Measuring the Mathematics Problem

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Thanks are extended to the Gatsby Charitable Foundation who sponsored the seminar; to those who attended and presented papers or contributed to discussions; and to Trevor Hawkes and Mike Savage for their efforts in producing the report.

Preface

Evidence is presented of a serious decline in students mastery of basic mathematical skills and level of preparation for mathematics-based degree courses. This decline is well established and affects students at all levels. As a result, acute problems now confront those teaching mathematics and mathematics-based modules across the full range of universities.

This report is aimed, therefore, at those in Higher Education who teach mathematics-based modules and those involved with admissions. There is a need for each department to become attuned to the full extent of the problem as it affects their students. The diagnostic testing of new undergraduates is recommended as an effective means of achieving this goal.

The report is also aimed at those charged with responsibility for setting the A-level Mathematics curriculum. Following the many changes introduced in the early 1990's we would suggest that now is the time to take stock.

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Summary

Findings

1. At least 60 Departments of Mathematics, Physics and Engineering give diagnostic tests in mathematics to their new undergraduates.

2. There is strong evidence from diagnostic tests of a steady decline over the past decade of fluency in basic mathematical skills and of the level of mathematical preparation of students accepted onto degree courses. Possible reasons for this include:
   - changes in GCSE and A Level syllabuses and structures;
   - greatly reduced numbers taking A Level Mathematics and Further Mathematics;
   - changes in the teaching force and in society;
   - lack of practice and poor study skills.

3. There is an increasing inhomogeneity in the mathematical attainments and knowledge of students entering science and engineering degree programmes. Factors here include:
   - insufficient candidates with satisfactory A-level Mathematics grades for the number of degree places available;
   - the freedom of A-level students to choose Statistics as an alternative to Mechanics, a choice that provides much less reinforcement of the pure mathematics.

4. The decline in skills and the increased variability within intakes are causing acute problems for those teaching mathematics-based modules across the full range of universities.

5. Diagnostic tests play an important part in
   - identifying students at risk of failing because of their mathematical deficiencies,
   - targeting remedial help,
   - designing programmes and modules that take account of general levels of mathematical attainments, and
   - removing unrealistic staff expectations.

6. Diagnostic testing should be seen as part of a two-stage process. Prompt and effective follow-up is essential to deal both with individual weaknesses and those of the whole cohort.

7. There is a need for greater institutional awareness of the cited problems and for adequate resources to deal with them.

8. The decline in basic skills is not the fault of the teachers in schools. Compared to their predecessors, they have to deliver a different curriculum, under very different and difficult circumstances, to quite different cohorts of students. From primary through to tertiary education, teachers share the common goal of inspiring young people to take mathematics seriously and to learn it well. They need to work together and to support each other in providing students with a coherent curriculum and a positive mathematical experience throughout their education.

9. There is also a need for the mathematics community, and its stakeholders in the broadest sense, to speak with one voice, to be heard in decision-making processes, and to represent its views effectively to the public through the media. A great step forward would be the creation of a small but representative Standing Committee for Mathematics which could act as a two-way conduit between the mathematics community and the bodies taking decisions affecting mathematics and mathematics education; in particular, the Committee would be given very early notice of possible government initiatives concerning mathematics and would be in a position to offer authoritative and representative advice to government and senior civil servants.

Recommendations

1. Students embarking on mathematics-based degree courses should have a diagnostic test on entry.

2. Prompt and effective support should be available to students whose mathematical background is found wanting by the tests.

3. The possibility of setting up a National Centre for Diagnostic Testing should be closely looked at. It would offer a flexible test-creation service with a broadly-based and easily-accessible database of questions to a wide range of educational institutions involved in the teaching and learning of mathematics.

4. In order to develop and monitor a mathematical education that is (i) appropriate to the needs of the 21st century and (ii) fully integrated from primary through to tertiary levels, the Government should establish a Standing Committee for Mathematics to include representatives from all sectors and stakeholders.

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Contents

1. Background and Outline of Report

This past decade has seen a serious decline in students' basic mathematical skills and level of preparation on entry into Higher Education. At least 60 Departments of Mathematics, Physics and Engineering now give diagnostic tests in basic mathematics to their new undergraduates. This decline presents a major problem in mathematics based degree courses right across the Higher Education sector — one which is widely acknowledged.

The problem has become steadily worse since the mid 1990's when several reports drew attention to the emerging "mathematics problem" at the sixth form/ HE interface and the inevitable consequences for students entering undergraduate courses in science and engineering; Institute of Physics (1994), Engineering Council (1995), Institute of Mathematics and its Applications (1995), London Mathematical Society (1995). In "Tackling the Mathematics Problem" it was pointed out that (i) Mathematics, Science and Engineering Departments appear unanimous in their perception of a qualitative change in the mathematical preparedness of incoming students — even among the very best and (ii) students enrolling on courses making heavy mathematical demands are hampered by a serious lack of essential technical facility — in particular, a lack of fluency and reliability in numerical and algebraic manipulation and simplification.

It was to accumulate evidence for the extent of this decline and its effect on first year undergraduate teaching and learning that a Gatsby Seminar was held on the 10th and 11th May 1999 at the Møller Centre in Cambridge. The 35 participants were mainly practitioners in Higher Education but also included representatives from the Qualifications and Curriculum Authority, Mathematics in Education and Industry and the Gatsby Technical Education project.

The Seminar ranged over the following related themes:

- The evolution of Mathematics A-level and other post-16 qualifications; the current situation in schools and colleges.
- Diagnostic testing in theory and practice.
- Using diagnostic tests to identify individual students' learning needs, to alert their teachers, and to inform module and curriculum design.
- How best to provide additional mathematics learning support; especially remedial help, Communications and Information Technology (CIT) solutions.
- The broader national perspective on teaching and learning and the role of the mathematics community.

As a result the report falls naturally into three main sections and a summary;

Mathematics at the Interface (1960-2000) which maps out the many changes to A level Mathematics and their effect on HE over the past four decades.

Diagnostic Testing in Theory and Practice examines the testing procedures in use at the Universities of Coventry and Warwick. They provide evidence of a steady decline in mathematical capabilities over a decade and illustrate that the decline is affecting even those departments who admit students with very high grades (29 or more points at A level).

Provision of additional support identifies a range of follow-up strategies by which University Departments are trying to cope with the problem of mathematical deficiencies. There are no simple solutions in this area. Various avenues are being explored and best practice has yet perhaps to emerge.

Summary consists of a number of key findings and four recommendations: They may be seen as a way forward in tackling the Mathematics Problem.

2. Mathematics at the Interface (1960-2000)

Mike Savage, Ann Kitchen, Ros Sutherland, Roger Porkess

Consisting of four presentations, the opening session aimed to set the scene for the seminar by reviewing (i) the changing pattern of the A level Mathematics Curriculum over four decades and (ii) the effect of this and other changes for students entering mathematics-based degree programmes in Higher Education. An outline of the key developments is given in Appendix 1.

A Level Mathematics: The Golden Age

Attention was drawn to the 1960's when A level was an entrance examination controlled by the universities and geared to serve their needs. A level Mathematics consisted mainly of Pure and Mechanics with the latter providing ample scope for the use and practice of algebra, trigonometry and calculus. With the benefit of hindsight this is now seen as a "golden age" for A level Mathematics in which able sixth formers, aiming for university, were inspired and stretched by a very talented teaching force. Students acquired the all important study skills together with sound mathematical knowledge and understanding which prepared them well for Higher Education.

A Level Mathematics: Changes to the Curriculum

The first major change to the curriculum appeared in the mid 1970's with the emergence of statistics. Students entering the sixth form now had a choice of mathematics courses; (i) Pure and Mechanics, (ii) Pure and Statistics, (iii) Pure and Applied (where the latter included a combination of topics in Applied Mathematics). This departure from homogeneity in the applications of mathematics generated problems for those teaching mechanics in first year undergraduate courses — with the result that diagnostic tests and/or additional support for mechanics were introduced in many universities. For the following decade, science and engineering departments learned to cope with students having varied backgrounds in Applied Mathematics. Indeed they were able to do so because Pure Mathematics at A level remained solid and students continued to be generally well prepared with regard to study skills, problem solving skills and basic mathematical capabilities.

The pace of change quickened in the mid 1980's. First, Universities lost control of the curriculum with SEC, SEAC, SCAA and now QCA validating courses. The Government gave the green light for curriculum development and a series of new, modular-type A level Mathematics courses were designed and trialled. A level was now viewed as a finishing post at 19 as well as an entrance examination for universities. The GCE examination was replaced by GCSE which, for Mathematics, brought a decline in students concept of proof and in their technical fluency and understanding of algebra. At a stroke A level Mathematics was undermined in a key area from which it has not yet recovered!

New A level Mathematics courses came on stream in the early 1990's with several new features including (i) a diminished pure maths common core (40% — reduced to 30%) (ii) a proliferation of modular syllabuses and (iii) a diversification of content.

Changes in A Level Mathematics : The Effect at 16-19

The period from 1986-1996 was identified as one of great change in A level Mathematics. By contrasting the teaching and learning situation pre 1986 with that post 1996 several key points are seen to be relevant to the universities perception of declining standards, Appendix 2. Class size for A level Mathematics, pre 1986, had a mean of around 10 and a maximum of about 15 with most students taking 3-4 science or maths A levels and going on to study Science, Maths or Engineering in HE or out to work. By 1996, class sizes had significantly increased with a mean of between 15 and 18, Mathematics was being increasingly used as a third subject for Arts, Science and the Humanities, and Further Mathematics had declined, by 50%, Hirst (1991, 1996). Only a third of students with A level Mathematics now went on to read Mathematics, Science or Engineering and only 40% of Engineering students had any A level Mathematics (UCAS 1996) Indeed some recruitment, particularly in the former Polytechnics, was of students from a vocational background with BTEC and subsequently GNVQ qualifications. With participation in HE having expanded rapidly in recent years it is apparent that many departments had recruited a number of less able students who would not have been admitted in the past. Referring to Kitchen (1999) it was suggested that the perceived decline in the mathematical capabilities of new undergraduates is partly due to (a) the changing A level cohort (b) able students choosing not to study mathematical subjects in HE and (c) the universities' policy on recruitment.

Changes in A Level Mathematics : The Effect on Higher Education

It is hardly surprising that the effect of substantial changes to the pre 19 mathematics curriculum would be felt across the whole university sector. Initially anecdotal evidence of a serious problem in Higher Education began to emerge;

"students don't seem to have the basics!";

"some have top grades and are having difficulty with algebra and calculus!".

Widespread disquiet within the mathematics community in universities gave rise to a report "Tackling the Mathematics Problem" (LMS 1995). This drew attention nationally to the seriousness of the problem — particularly for HE — where students were now able to get good grades at 'A' level and yet be unprepared (by earlier standards) for mathematics-based degree programmes. At this time, many first year undergraduate mathematics teachers, across a range of universities, were reporting the emergence of a 'tail' of weak students (deficient in basic mathematical skills) — a tail which grew in size and gave rise to "high failure rates". By the late 1990's it was clear that this decline in basic mathematical knowledge, skills and level of preparation was a source of continuing concern for teachers of mathematics-based modules in HE. The need to introduce year-on-year changes to the curriculum and to provide remedial help for those with the more obvious deficiencies had become common place. Furthermore diagnostic tests, both written and computer-based, were increasingly being introduced by Departments of Mathematics, Physics and Engineering to assess what students could and could not do, as well as assess their overall level of preparation. In a recent report Sutherland and Dewhurst (1999) considered the mathematical knowledge of undergraduates as they enter university across a range of disciplines. This study concludes that the school curriculum is not adequately preparing students for a mathematics degree or for the mathematical components of other higher degree courses (Physics, Computer Science, Engineering...). The study also points out that University Departments are finding various ways to deal with the mathematics teaching of first year undergraduates. These include; the redesign of first year mathematics courses, the provision of remedial help, drop-in workshops and computer-based mathematics learning centres. It is further noted that the assessment of the effectiveness of the changes introduced, is difficult. For example the diagnostic testing and computer-based learning has not been systematically evaluated.

A Level Mathematics Teaching

A feature of the teaching profession over the past 30 years is the loss of status of teachers, and especially maths teachers. As a consequence, the composition of the teaching force has changed as (i) potential and able recruits have been put off the profession, not least, by talking to existing teachers who have become demoralised, and (ii) many older teachers have felt that retirement is preferable to having their work unappreciated or indeed castigated.

The need to recognise the extremely difficult nature of the job expected of today's A level Mathematics teachers was strongly emphasised. Compared to their predecessors, they teach a much changed and now varied curriculum to students with a quite different motivation and background under very different circumstances (large class sizes and inhomogeneous cohorts of students). Indeed such teachers need to be supported and encouraged if we are to stand any chance of attracting able recruits into the profession. It was further pointed out that most of the competent maths teachers in our schools and colleges are now in their 40's and 50's. As they retire they are not being replaced by young teachers of anywhere near the same calibre. Even good schools are finding it almost impossible to recruit acceptable new mathematics teachers. Many posts are filled mainly by people who are not mathematicians and do not have a sound background in the subject. Consequently trying to maintain, let alone improve, the quality of students leaving school for Higher Education is rather like running up a down-going escalator — and one which, if anything, is accelerating. Whatever problems universities now face are likely to get worse rather than better over the next decade.

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3. Diagnostic Testing in Theory and Practice

John Appleby, Duncan Lawson, Tony Croft, Trevor Hawkes,
Douglas Quinney, Brian Sleeman

Diagnostic tests are a direct response to the perceived decline in the mathematical knowledge and skills of new students in relation to the mathematical requirements of their programmes of study. The majority of tests, operating in Diagnostic Mode, are designed to assess students' prior mathematical knowledge and skills and so allow their teachers

- to identify individual strengths and weaknesses
- to identify overall deficiencies in the cohort
- to identify unrealistic staff expectations
- to inform module and curriculum design and delivery.

A second mode of operation is the so-called Drill and Kill Mode, designed for students on courses requiring high A-level mathematics grades. Students are able to identify particular weaknesses and areas where they are short of practice and are then required to take diagnostic tests repeatedly until they reach a prescribed standard.

Six presentations served to reveal a wide range of evidence of a decline in the mathematical preparedness amongst new undergraduates in Departments of Engineering, Mathematics and Physics from both the new and older universities (Appendix 3). The overall message was clear; the problem had become steadily worse, was now widespread and the A Level mathematics grade is not a reliable indicator of competence in basic skills.

Of these presentations, two are given in full — in the following sub-sections — in order to contrast the Diagnostic Mode of testing (at Coventry University) with the Drill and Kill Mode of testing (at the University of Warwick). The first reveals strong evidence of a steady decline in mathematical capabilities over a decade whereas the second illustrates how the decline is affecting those departments who admit the most able students (with 29 or more points at A-level).

31 Changes in the Competency of A Level Students

Duncan Lawson, BP Mathematics Centre, Coventry University

Introduction

The expansion of higher education in the 1980s created less homogeneity in cohorts of students entering university. Not only were students with lower A level qualifications than in previous years entering higher education there was also an expansion in the types of entry qualifications as vocational education (BTECs and latterly Advanced GNVQs), Access and Foundation courses became accepted routes into university.

It was against this background of inhomogeneity that Coventry University introduced its standard diagnostic test [1] in 1991. The purpose of this test was primarily to determine what incoming students could and could not do. The same test has been used every year since 1991. It consists of 50 multiple choice questions each offering 1 correct option, 3 (wherever possible, plausible) incorrect options and a Don't Know option. The test covers seven topics: basic arithmetic, basic algebra, lines and curves, triangles, further algebra, trigonometry and basic calculus.

Two sample questions are given below:

Basic Algebra — If x(x+1) = 6 then

A: x = 6 or x+1 = 6 B: x = 6 or x+1 = 1 C: x+2=0 or x-3=0
D: x-2=0 or x+3=0 E: Don't Know

Lines & Curves — The curves y = x2+2 and y = 3x

A: have no point of intersection B: intersect at the point (1,3) only
C: intersect at the point (2,6) only D: intersect at (1,3) and (2,6)
E: Don't Know

Students take the test, essentially without warning, during induction week. The aim of the test is to discover in which areas of mathematics students have immediate competence and in which they do not. Students are assured that no marks are given for the test and that the information gathered from it will be used for their benefit. They are told that guessing may obscure weaknesses that they need to correct and are encouraged to select the 'Don't Know' option rather than guess.

Each student receives an individual diagnosis outlining his/her strengths and weaknesses in the seven topics covered by the test. (Students are not given marks, instead they are given a qualitative description of their performance on each topic which is either 'Excellent', 'Satisfactory' or 'Some revision may be needed'.) Printed on each diagnosis is the advice that, where topics have been shown to need some revision, students should visit the BP Mathematics Centre for discussion with Centre staff about appropriate action to fill in gaps in background knowledge. The Centre has a range of materials available for students to use, either in the Centre or to take away and offers one-to-one support for students. The Centre operates as a drop-in facility, currently open for 25 hours per week. It is up to the students as to whether or not they take advantage of the help available in the Centre. There is no compulsion on them to visit. Currently the Centre receives around 1000 student visits per term.

In addition to the personal diagnoses, cohort summaries are also compiled. These cohort summaries are sent to course tutors to enable them to advise students in module selection and to module leaders to inform teaching approach (and starting point).

When the test was introduced in 1991 only a relatively small number of students took it. These were on courses (often engineering HNDs where a high proportion of the intake was from a BTEC background) where experience had shown that the entry cohort was most likely to struggle with mathematics. As time has gone by the number of courses using the diagnostic test has increased and now it is taken by all students entering courses containing a significant study of mathematics (including courses in the mathematical sciences). Currently the test is taken by approximately 600 students per year.

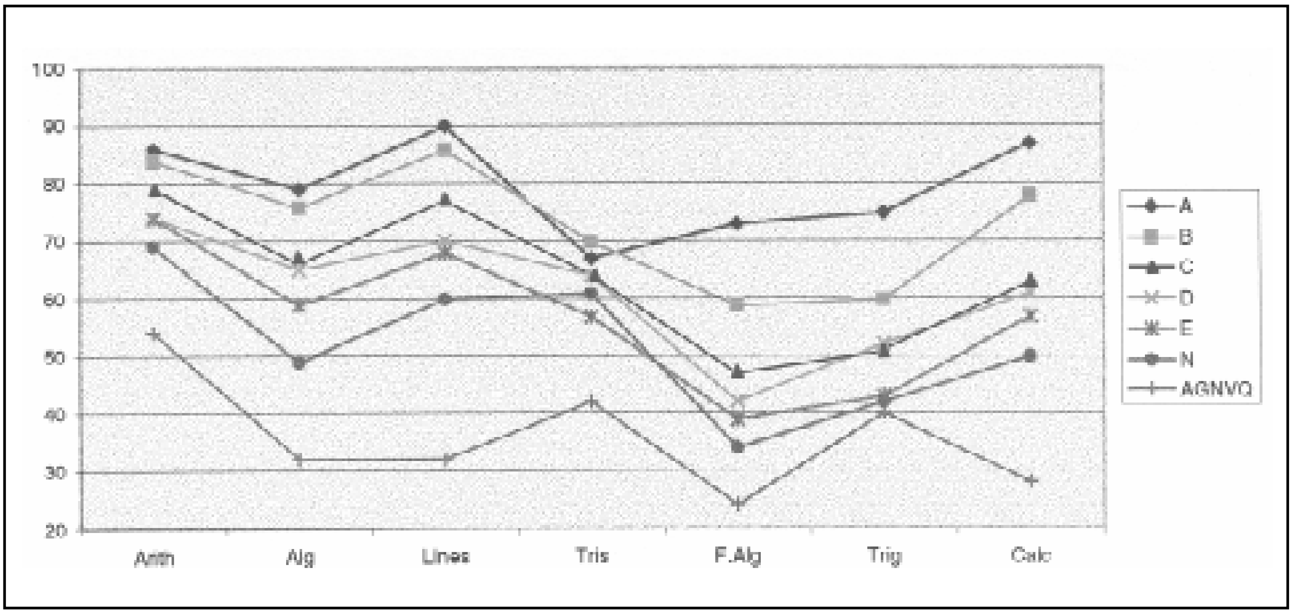
Results

In addition to the immediate information which the diagnostic test provides (and for which it was designed), by recording the results every year, a large database of student performances has been built up which enables a study of changes in performance over time to be made. Such analyses have been published for the data up to 1997 [2-4]. Here we extend this analysis to include the results of students entering higher education in 1998.

Figure 1 below shows the performance of the 1998 entry cohort for those students with A-level grades A — N and those students with Advanced GNVQ (AGNVQ). On each topic (except triangles, which it could be argued is really a GCSE topic) the performance decreases with A-level grade and the results of Advanced GNVQ students are significantly below those of students with A-level grade N.

For grades B to N the worst performance is in Further Algebra. The questions in this section cover function notation, the factor theorem, negative and fractional powers, expanding a cubic, the definition of logarithms, inequalities and partial fractions. Even amongst grade B students only just over half of these questions were answered correctly. This has serious implications for the exposition not only of university mathematics but also of many engineering topics.

Figure 1: Performance of the 1998 entry cohort



The results of the Advanced GNVQ students are particularly disappointing. The syllabus topics of the optional and advanced mathematics units in Engineering Advanced GNVQ are very similar to the syllabus topics in A-level. However, the results from the 1998 sitting of the diagnostic test indicate that these topics are not covered in anything like the same thoroughness. This has serious implications for many engineering courses. The latest edition of SARTOR [5] specifically identifies Advanced GNVQ as an acceptable alternative to A-level as an entry qualification for accredited degree courses. However the mathematical experience of Advanced GNVQ students is vastly different to that of A-level students.

Figure 2: A comparison of successive cohorts of Grade D students

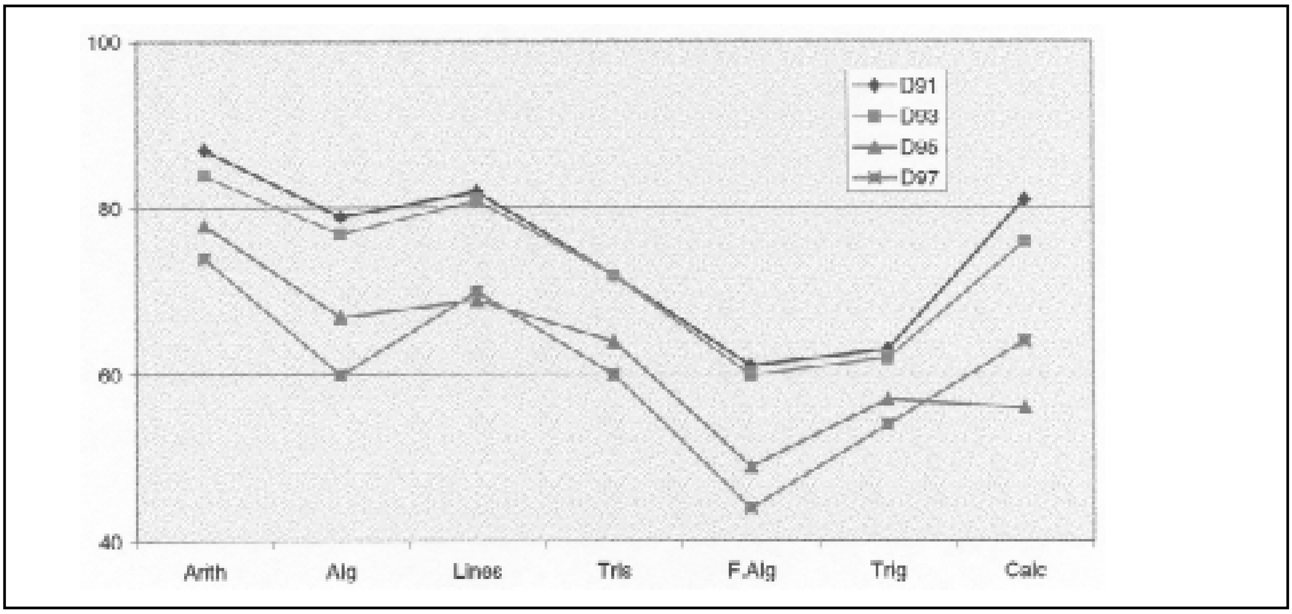
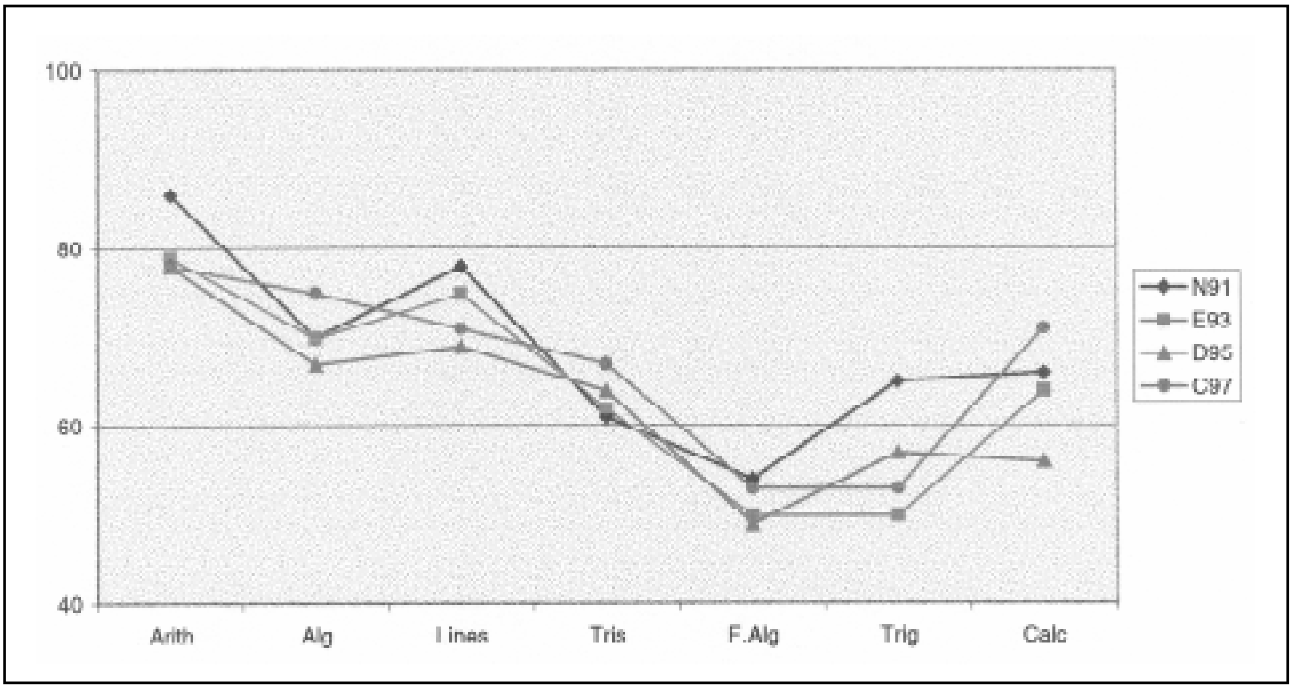


Figure 2 shows the performance over time of students entering with A level grade D. Grade D has been chosen for this analysis because there is more data for this grade than for any other. As mentioned above, when the diagnostic test was first introduced it was primarily given to those student groups thought to be most at risk and these groups had very few students with high A-level grades. In later years there is a reasonable amount of data for all grades.

The results for grade D students are typical of what happens at other grades. This gives clear evidence that there is a decline over time in the competency of students with the same A-level grade in the topics covered by the diagnostic test. The difference between the 1991 and 1993 cohorts was small, however there was a sharp drop in competency in the 1995 cohort and a further noticeable drop with the 1997 cohort. The performance of the 1998 grade D cohort is not shown in Figure 2 however it is similar to that of the 1997 cohort with a marginal improvement in basic algebra and triangles offset by slight decline in further algebra, trigonometry and basic calculus. It is interesting to compare the performance of different grade cohorts in different years. Figure 3 shows the results for the 1991 grade N students, the 1993 grade E students, the 1995 grade D students and the 1997 grade C students.

Figure 3: A comparison of different grade cohorts from different years



Comparing these four cohorts we see that the N91 cohort performs best in four of the seven topics and the C97 cohort performs best in the other three. However, there is no significant difference between the overall performance of these four groups. This suggests that, in the skills which the diagnostic test assesses, there is a 'grade dilution' of approximately one grade every two years.

The overall score in the test of students with different A level grades provides some startling information. Table 1 gives the overall scores (out of 50) for the 91, 97 and 98 entry cohorts (grade A has been omitted because the sample size in all years is too small to provide any useful information). Comparing the results of the 1991 and 1998 cohorts we see that for grades C to U the score drops by, respectively, 7.8, 6.7, 7.4, 8.7 and 7.3 (out of 50) or roughly speaking one mark per year.

Table 1 also shows what we have seen in Figure 3, namely that the performance of the 1991 grade N cohort (score 34.4) is very similar to that of the 1997 grade C cohort (score 34.0). Whilst it cannot be deduced from this that 97 Grade C students would have failed in 1991 (as was sensationally claimed in the Sunday Times [6]) it is strong evidence that there are major problems in terms of the mathematical skills traditionally required by university courses. With the 1998 grade C cohort the situation is even worse. These students scored on average 2.3 marks (4.6%) less than the 1991 grade N cohort. It appears that the decline continues.

	1998		1997		1991	
A level Grade	Score (max 50)	Sample Size	Score (max 50)	Sample Size	Score (max 50)	Sample Size
B	36.8	27	36.4	17	40.5	2
C	32.1	52	34.0	48	39.9	12
D	30.6	58	30.3	56	37.3	33
E	28.2	46	30.3	48	35.6	63
N	25.7	22	27.8	17	34.4	28
U	22.8	9	23.8	19	30.1	28

Table 1: Diagnostic Test Scores (by A level grade)

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Further Evidence of a Decline

Just prior to the publication of this report information came to light on the diagnostic testing of undergraduate physicists at the University of York, where Professor J Mathew has used the same mathematics test to collect data over a twenty year period 1979-1999. Originating from a Physics Interface Project, this test was designed to assess basic mathematical knowledge and skills. Two sample questions are presented in Appendix 4 along with a graph showing the performance over time of different cohorts with similar A level grades. This performance remains more or less constant until 1990 when there is a sharp fall followed by a steady decline over the past decade — thus confirming the trend identified by Dr Lawson.

3.2 Diagnostic Testing in the Department of Mathematics

Trevor Hawkes, Department of Mathematics, University of Warwick

Academic Background

The Department admits 220 single-honours mathematics students with an average score of 29.5 on their best three A-Levels. All have taken a STEP or Special Paper in Mathematics and most have passed this at Grade 2 or higher. Furthermore, the majority have taken double Maths at A-Level. In addition, the Department teaches the first-year Mathematics core courses to 150 joint-honours degree students based in the Departments of Physics and Statistics. These students must have an A grade in A-Level Mathematics and enter with an average of 27 A-Level points; however, they are not required to take a STEP or Special Paper.

The Department offers rigorous three-year BSc and four-year MMath programmes including a thorough grounding in traditional Analysis in the first year. The Mathematics core has four main sequences: Algebra, Analysis, Geometry & Topology, and Applied. The programme offers a high degree of flexibility and, after their first year of study, students may present themselves for examination in a wide range of options from both within and without the Science Faculty. There is little space in the curriculum for remedial courses.

Rationale for Warwick's 'Drill-and-Kill' Approach

During the 80s, members of the Department in the front line of undergraduate teaching became convinced of a steady decline in the basic mathematical skills of their new intakes. The students arrived less able to manipulate algebraic and trigonometric equations and inequalities with speed and accuracy, and less proficient at the tasks of basic calculus, such as differentiating and integrating elementary functions. During this period of decline, the average A-Level scores of our intakes went up.

The Department responded in two ways.

- The syllabuses of first-year courses were rewritten to reduce the steepness of the initial learning curve and provide a gentler transition from the more concrete and prescriptive style of A-Level mathematics to the more conceptual and open-ended approach at university.
- Optional classes were offered to first-year students to help them plug any gaps they identified in their A-Level skills.

These classes ran for several years in different formats but were always a failure. After a couple of weeks, attendance would drop to a mere handful. The classes quickly became negatively seen as 'remedial', and the fact that no course credit was attached to them gave the impression that the Department did not regard them as important.

In the mid 90s, a raft of more radical solutions were introduced: weekly marked homework for exam credit in all core modules, additional small-group teaching (three hours a week in squads of four), a small-classroom approach to teaching Analysis, group work, IT-based enhancement of learning. And the optional classes designed to bring students up to speed in their A-Level skills were replaced by diagnostic tests in five skill areas on arrival, followed by weekly resit tests until a score of at least 80% is achieved. A small amount of exam credit is attached to each of these tests (amounting to 0.4% of the overall first-year mark, which in turn contributes only 10% to the final degree score) but this provides an effective inducement to full participation and widespread success. The five A-Level skill areas together are called *Maths Techniques* and form a small part of the first-year bridging module *Foundations*.

The Diagnostic tests

- give students immediate feedback of how they measure up, on arrival at university, to the standards expected of them in basic A-Level skills,
- give tutors (who mark the Diagnostic tests) an instant snapshot of the level of basic skills of the new students they will be teaching,
- and allow the Department to monitor the changing background abilities of its incoming students over the years.

The weekly resit tests for students who fail to reach the threshold of 80% in one or more of the skill areas in the diagnostic tests, and the regular reporting of students' progress to their tutors, send a clear message that the skills are important to further progress in the degree course. The very modest amount of exam credit that is awarded for success reinforces this message and provides the incentive for students to develop the accuracy and skill required to achieve the 80% target.

The Logistics

In late August, as soon as the Department receives the names and addresses of the admissions candidates who have achieved its offer, a letter is sent to them explaining that there will be a diagnostic test in five skill areas at the start of term. This mail shot includes a booklet of practice questions, together with advice on how to deal with them and references to the material in standard A-Level texts. Students are advised to spend the month before they come to Warwick getting up to speed on these skills and filling in any gaps they may have in their A-Level knowledge (for example, complex numbers).

On the first Wednesday of the new academic year, all first-year single-honours and joint-degree Mathematics students sit a 2-hour diagnostic test covering four skills areas: Differentiation, Integration, Trigonometry, Inequalities. The tests are held in normal lecture theatres, have the format of question-and-answer booklets. These are marked by the students' personal tutors and returned to them the following week at the first weekly tutorial, when any shortcomings are identified and discussed.

Students who fail to reach the 80% threshold in one or more of the four skill areas, can take resit tests (held at time-tabled slots during the subsequent six weeks) as often as they like until they achieve this percentage.

To minimise marking costs, the resit tests are staged in the following format:

- Before the test begins, each candidate collects a blank answer sheet to which is attached a second carbonised copy and takes a seat in the lecture theatre.
- At the start of the test, the questions are displayed on a screen using an OHP.
- At the end of the 30-minute test, students hand in the carbon copies of their answer sheets.
- The answers are then displayed on the screen and students mark their own papers. They can raise questions about the validity of their answers at this stage.
- Students who claim to have scored 80% or more hand in their marked top copies. These are then checked against the carbon copies and a sample are remarked.

Under this arrangement, only a fraction of the successful tests are marked; no time is expended marking the tests of the unsuccessful candidates.

A fifth skill area, elementary algebraic manipulation, is tested by computer using a modified version of John Appleby's assessment program Diagnosys, which was originally designed to provide diagnostic testing of Engineering students at the University of Newcastle. With Appleby's permission, Warwick added libraries of questions to enable Diagnosys to generate unlimited random tests, a timing mechanism, and an email system for sending progress reports to students' tutors. To obtain the exam credit for this fifth skill area, students must pass the

40-minute Diagnosys test at the 90% level but are allowed to retake the test as often as they wish. A secondary purpose of Diagnosys was to encourage students to get logged on to the University network at an early stage of their course, but with the steady growth of IT in the degree programme, this is now a superfluous motive.

Success in one of the five skill areas is rewarded with a contribution of 4% to the final overall score in the first-year bridging course Foundations. Thus up to 20% of the final Foundations mark comes from the 5 Maths Techniques skill areas but contributes only a maximum of 0.2% to the final degree mark.

Students who do not achieve the 80% threshold in one of the written tests by the end of Week 7 or the 90% target in the computer-based Diagnosys test by the end of Week 10, are given a score of zero (instead of 4). This unforgiving marking system stresses the importance of both accuracy and speed to succeed.

Learning Support

The August mailing of practice test questions includes advice and references to background material in A-Level textbooks.

Tutors are expected to offer help to those of their tutees who appear to have difficulties with the Maths Techniques material.

From Week 3 onwards, regular help classes in the various skill areas are provided for students who are having trouble passing the resits. These classes are taught by a local retired school teacher.

Much of the first-year teaching and learning takes place in squads of four; these are established at the start of the academic year and stay fairly stable throughout the first two years of the degree programme. These squads are encouraged to help each other (when two or three in a squad have already passed a given skill area, they are urged to coach the one or two that haven't).

Detailed back-up notes are available for the computer test Diagnosys.

Conclusions

The approach described above provides an effective and low-cost method of raising the level of basic mathematical skills in beginning undergraduates. Over 350 students from three departments are put through the system within a 10-week term for under £1000. The great majority of these achieve the desired 80/90% thresholds in all skill areas within this time. After such a system has run for two years or more, the tests and related materials can be recycled without risk of compromising security.

The high entry standards into the Warwick degree course ensure that the great majority of the students are able to raise their performance in Maths Techniques with the minimum of support and individual attention. It may not work as well on students with a weaker or more variable mathematical backgrounds.
In due course it is hoped to convert the Maths Techniques resit tests into a computer-based format. This would enable the tests to be used in both formative (practice) and summative (examination) modes. After the initial investment, this would reduce costs still further, and would bring the other advantages of Web-based assessment:

- easy monitoring of individual progress;
- analysis of individual and collective weak areas;
- on-line help;
- quick reporting and feedback to students and staff alike.

Problems with (i) displaying mathematics on the Web and (ii) examination security still need to be solved.

4. Provision of Additional Support

A Summary of Reports from Working Groups

Once the results of a diagnostic test are known the question arises as to what we do with the results of the test and , in particular, "how do we provide effective support for those students with deficiencies in basic knowledge and skills?".

It was clear from the collective experiences of the seminar participants that there is no simple solution- no panacea- to what has rapidly become one of the most challenging problems at the school/university interface. We are at the stage where a number of individuals in various institutions are currently exploring a wide range of follow-up strategies. These include supplementary lectures, additional modules, computer assisted learning, Mathematics Support Centres, additional diagnostic tests and streaming. Each has its merits and brief details are given below.

Supplementary Lectures

One of the first attempted remedies was to introduce a series of supplementary, remedial lectures running in parallel with the first year course. There are two problems here; (i) the need for students to give sufficient time and attention in order to master the remedial material and (ii) the danger of overloading weak students with additional studies when they are probably struggling with other modules.

Additional Assessed Modules

Some departments require their weaker students to rectify their deficiencies by taking additional mathematics modules alongside their main course. Once again the danger is to overload students with too much additional work. As one participant remarked "can a student lacking basic skills cope with the demands of the current mathematics and engineering modules whilst attempting to repair the earlier damage?".

Lower Level, Transition Modules

There is a clear need to circumvent the problem of overload whilst recognising that weak students do need a relevant transition module to make up for shortfalls in their background. A strategy used in some institutions is to replace mainstream mathematics modules by transition or bridging modules which permit students to get to grips with lower level material at a slower pace and in a less diverse group. The mainstream mathematics modules are then taken later in the programme.

For weak students the downside of this approach is that their mathematical experience remains out -of- step with the mainstream giving rise to other problems

- discovering that they have not covered the mathematics required for their engineering modules.
- discovering that their options are severely restricted.

Computer Assisted Learning (CAL)

The increasing availability of student centred computer-based support material now presents an ideal opportunity to follow up a diagnostic test and address the problem of mathematical deficiencies using CAL.

Given a diagnostic profile of students mathematical knowledge and skills (set against pre-determined objectives in Mathematics) it is now possible to provide automatic links to CAL material such as that developed under TLTP, for example

- Mathwise; 50 computer based modules developed by the UK Courseware Consortium
- Calm; computer assisted learning in Mathematics, Heriot Watt University
- Metric; TLTP project based at Imperial College, London
- Transmath; TLTP project based at Leeds University
- Calmat; computer assisted learning in Mathematics based at Glasgow Calendonian University.

This approach is being pioneered at Keele University with Dr D. Quinney where the aim is to provide support material where a central feature is a symbiotic relationship between computer based material and tutorial support.

Mathematics Learning Support Centres

Several universities have established a Mathematics Support Centre to enable students experiencing difficulties with mathematics to obtain supplementary resources and tuition. The aim is to devise follow-ups to the diagnostic test which are tailored to individual needs and, since the centre is open-access, students can attend at any time. Such centres can be very effective in dealing with students having specific isolated difficulties. However, for the less well-prepared students a drop-in surgery is hardly sufficient. If they are to gain a coherent body of mathematical knowledge there is no substitute for an extended and systematic programme of study.

Additional Tests

Diagnostic tests reveal weaknesses in basic mathematical knowledge even amongst the most well qualified students.

At the University of Warwick a successful method has been devised for dealing with deficiencies with very able students having over 29 A-level points and most with Further Mathematics — see Section 3.2, Diagnostic Testing in the Department of Mathematics: Dr T Hawkes.

Mixed Ability Teaching vs Streaming

A diagnostic test will identify both the strengths and weaknesses of individual students and also the range of abilities of students in the cohort. The question then to be addressed is how best to teach them in the first semester mathematics module? Often course requirements and financial considerations are such that students have to be taught as a single group and so we are engaged in mixed-ability teaching!

There is an increasing awareness, however, amongst first year teachers of mathematics that mixed ability teaching, of a first year cohort with diverse knowledge and abilities, is quite inappropriate . Indeed it can be disastrous by further compounding rather than solving the initial problem. The weaker students are “at risk” and remain so without a carefully planned programme of help to assist them on the road to recovery.

At the University of Leeds an experiment is currently underway with the teaching of mathematics to first year physicists who are admitted with A level mathematics grades A to C but also includes a number with grade D(and a good grade in physics). Diagnostic testing identifies students who are “at risk”-in the sense that they are the ones who are most likely to fail the module when taken by the whole cohort. In an attempt to help this weaker group the cohort has been streamed with those “at risk” taught separately. They will cover the same material (including A level revision material) at a slower pace over a longer time and sit the same examination as the “standard group”. By the end of the academic year we shall know the extent to which the experiment has proved successful!

Finally we conclude with a perceptive remark from one seminar participant on the effect of follow-up strategies which puts into sharp focus the extent of the challenge that now faces us;

> “ the problem of declining mathematical ability is a long standing one and its roots go deep. It is not at all clear that such cosmetic exercises at the point of entry to tertiary education will be sufficient.”

Findings and Recommendations

These are given in the Summary on page (iii).

Appendix 1

Mathematics at the Interface (1960-2000)

1960s	Advanced level GCE (A-level) was an entrance examination for the Universities — run by them and geared to serve their needs. A-level Mathematics was mainly Pure and Mechanics.
1970s	Emergence of Statistics. Universities experience problems teaching Applied mathematics at Year 1. Diagnostic testing of mechanics.
1980s	Universities lose autonomy in curriculum to the Government. SEC, SEAC, SCAA and now QCA validate awards. Curriculum development pre A-level of GCE and CSE into GCSE. Numbers taking Mathematics at A-level start to drop dramatically.
1990s	Deficiencies with algebra become apparent. New Mathematics courses emerge (1992) These try to make mathematics a) more popular, and b) more accessible There is a wide variety in the mathematics content and mastery of the many different A level syllabuses available. Many students taking A-level mathematics do not go on to use their mathematics in H.E. Chaos/confusion in H.E. Reports and analyses are carried out by various bodies. Development of diagnostic testing of basic skills within universities.

Appendix 2

A level mathematics: the post 16 perspective.

When	Pre 1986	1986+	1996+	2002+
Defined by	• Examination boards • Led by Universities • No statutory core	• Government control. • Mathematics core • 40% Pure Maths (less algebra)	• Need to increase accessibility • New core • 30% Pure, 10% Stats 10% modelling	• New core 50% Pure, 25% applications
	All Linear ⟷			All Modular
Taught to	• Steadily increasing numbers • Class size rose to a mean of around 10 and max of about 15 • Most university entrants had A levels	• Failing numbers • Increase in class size • Maximum in FE rises to 25+ • Growth in other maths qualifications	• Numbers start to improve. • Class sizes continue to increase. • Mean now between 15 and 18, • Continuing diversity	?
	Mainly 11-18 schools ⟷		Mainly post 16 institutions	
To whom	• Most taking 3 or 4 science or maths A levels • Most going on to science/maths/ engineering or out to work	• Growth in use of Mathematics as a third subject for Arts, science and Humanities • Decrease in Further Maths	• Only $^1/_3$rd of students with A level Maths go on to read science, engineering or maths • only 40% of engineering students have any A level maths (UCAS 1996) • Decrease in emphasis on Pure Maths	?
	Mainly science A levels ⟷			Mixed A levels
Other factors	• Very little outside work • Most students full time • Maths sex ratio around 2:1	• Increase in numbers of students doing outside work • Advent of AS Further Maths may have less Pure content • Sex ratio is 62%,38% in 1995	• The mean no. of hours of paid employment per week is around 12. (study of 3 large post 16 institutions, 1997) • Push for more AS League tables No change in sex ratio	?

Ann Kitchen, Centre for Mathematics Education, University of Manchester,

Appendix 3

Reviews of Diagnostic Testing

3.1 Gatsby Seminar on Diagnostic testing in mathematics experience from the Universtiy of Newcastle

JC Appleby, Department of Engineering Mathematics

Introduction

Diagnostic testing has been used at foundation level and then at first year level in mathematics at Newcastle for eight years. For several years a basic mechanics test has also been in use. Diagnostic testing on entry may be used to: select students for special treatment or streaming, to identify specific areas needing attention for each individual and to survey group knowledge to inform course design and delivery. It is also interesting to discover whether such tests have any predictive power with regard to future success.

Our overall aims in using diagnosis and remediation were to assist every individual with information about their own needs, to provide sufficient support for the weakest to enable them to catch up and progress to a degree, and to inform curriculum design and admissions policy by relating knowledge to background qualifications and to subsequent progress.

Results for the Mathematics test

The Mathematics test results are not in themselves very startling, revealing almost complete knowledge of the most basic skills, and a predictable patchiness on more advanced ones, even those theoretically covered by some higher GCSE syllabuses (Table 1). The possibility of entering algebraic answers to questions, and the correspondingly very limited use of multiple-choice questions, helps to produce results that match one's expectations. However, all teaching staff need to be aware of the significant proportion of students not knowing, for example, how to factorise difference of squares or how to divide algebraic fractions. Moreover, a lack of knowledge at this level is likely to mean not that they haven't met the topic, but that they have failed to absorb it reliably, and that therefore they may not pick it up quickly when revised.

Table 1. Knowledge of selected basic skills by Stage 1 Engineering students 1997 entry — proportion knowing, or inferred as knowing, each topic:

Manipulation of positive powers	91%
Manipulation of negative powers	85%
Manipulation of fractional powers	44%
Expansion of two brackets, collecting terms	92%
Factorising difference of two squares	55%
Dividing algebraic fractions	68%
Definition of radians	68%
Gradient of line through two points	77%
Equation of circle, given radius and centre	16%

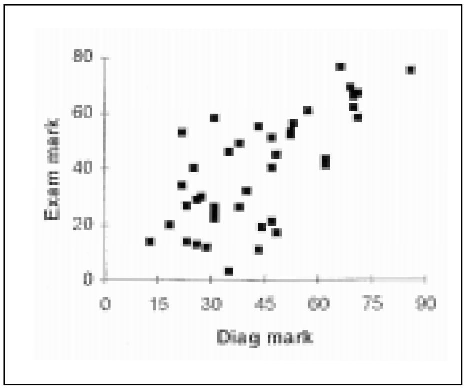


Figure 1. Relationship of diagnostic maths mark and first maths exam mark, Stage 0

Figure 1 shows the correlation of diagnostic mark with a later exam mark. This is of interest in itself, but also provides some validation, which is needed to use a diagnostic test with confidence, particularly one with inferences about knowledge based on the skills network. For the 1996/7 Foundation Year, the correlation is high (for this kind of data) at 0.72, comparing well with the correlation between first and second exams in mathematics. However, note that the predictability for an individual is measured by adjusted R-squared — the proportion of the variance in the exam mark accounted for by the diagnostic mark. At a value of 0.50, this is high (as predictions go) but indicates the difficulty in assigning remedial tuition — perhaps half of the students so assigned will be in the wrong group!

At Stage 1, the correlation (0.61) and prediction (adjusted R-squared = 0.37) is less good, reflecting perhaps the less suitable coverage of the Test; the first semester maths course is largely a revision of A-level in any case.

An additional statistic is provided by the relationship of departmental means of diagnostic and first exam marks. Five departments are split into three lecture groups, and many tutorial groups, and we were interested in the effect of the different lecturers, together with departmental course loadings, pastoral factors etc. The correlation of the average exam mark for the departments with the average diagnostic mark was a remarkable 0.994 in 1996/7. That is, the departmental performance in the first maths exam in January could have been predicted almost perfectly from the diagnostic marks available in early October!

Remedial provision

As mentioned above, at Foundation Year and at Stage 1 in some degrees, a full remedial module is provided for those deemed to be at risk, based on their diagnostic mark and qualifications. The two years' results available for Stage 1 students show a pleasing improvement relative to the main group in their first exam, but no subsequent improvement

Table 2. Remedial group results — Stage 1 entrants to Mech & Mats degrees

		Stage 1 results			Stage 2 results		
Year	Number	1st exam	2nd exam	All subjects	1st exam	2nd exam	All subjects
1994/5							
Remedial	13	52	43	54	49	43	53
Whole	95	52	51	53	55	47	52
1995/6							
Remedial	10	54	42	47	50	n/a	n/a
Whole	78	56	50	52	63	n/a	n/a

(Table 2). The Stage 2 results exclude those students who failed or dropped out in Stage 1 (three and four respectively), but the mean results are still below the rest of the group (some of whom had also gone).

The DIAGNOSYS Test

DIAGNOSYS is a skills-based computer testing system for basic mathematics. It is supplied with questions on 92 skills in numeracy, algebra, trigonometry, statistics etc. Normally a student will be tested on 50-70 skills by selecting certain topic areas. The knowledge-based approach used allows diagnosis of this range of knowledge by asking questions selectively, responding to background qualification and to previous answers. For example, consider two algebra skills:

Expand x(x+l) => Expand (x+1)(x+2)

Knowledge of the second implies knowledge of the first, whilst ignorance of the first implies ignorance of the second. Typically, fewer than half the skills need be tested, and a test of c60 skills can be completed in one hour. The skills are categorised as being at four'levels' of difficulty, and are arranged in a hierarchical network.

The output files are processed to produce individual 'profiles', ranked listings, 'group profiles' of knowledge (including analysis of sub-groups, such as different qualifications, sex, etc.), and tabulated answers to reveal common misunderstandings.

Using a hierarchy of skills thus has a twofold benefit: it reduces the number of questions asked, and allows the system to target questions more appropriate to the user. The network, inferences and question design have been validated in several ways, as described in Appleby, Samuels and Treasure Jones 1997.

DIAGNOSYS is designed to permit minor, major or complete changes to the questions and knowledge-base. Tests on other subjects have been constructed by others using this system.

Publications

1997 DIAGNOSYS — a knowledge-based diagnostic test of basic mathematical skills. Computers in Education, 28, 113-131 (with PC Samuels and T Treasure Jones)

1997 Diagnostic testing of mathematics and mechanics on entry Mathematical Education of Engineers, Loughborough, IMA, 1997. (with A Anderson)

1995 Managing the transition to University education: computer-based diagnostic testing of background knowledge. International Congress of Engineering Deans and Industry Leaders, Monash, 1995. (with A Anderson and others)

1995 Diagnosis and treatment of students from varied backgrounds. IMA Conference: The Mathematical Education ofEngineers, Loughborough, 1994. (with A Anderson, P Fletcher)

3.2 Mathematical preparedness for entrance to undergraduate engineering degree programmes, and diagnostic testing

Dr A Croft, Department of Mathematical Sciences, Loughborough University

In 1996 a Mathematics Learning Support Centre was established at Loughborough University. Its primary raison d'etre was to support significant numbers of engineering undergraduates who were having serious difficulties coping with the mathematical demands of their programmes. Since then it has developed an exceptionally wide range of resources using a mix of traditional and modem technologies and now offers its services to any undergraduate student in the university who might benefit from additional help with learning mathematics.

The Centre has been the focus of a number of initiatives including diagnostic testing of new students. This has been carried out for three years now (1996, 7 and 8) and was implemented as follows:

(i) for practical reasons the test was paper-based multiple choice (OMR marked). About 600/700 students are involved, and to test such a large number by computer within a couple of days of arriving at university is impossible.

(ii) it has only been possible to run a one hour test covering number, basic algebra, and basic calculus very superficially. This is because the test either had to be fitted into a busy induction period time-table or, exceptionally, a first maths lecture. Further, the questions on the test were thought by staff to be rather simple, and so getting a question correct does not imply competence. Certainly getting questions wrong raises concerns. Those of us concerned with the test recognise its inadequacies and ideally would require students to do a much more comprehensive and longer test.

(iii) Students and tutors are given feedback within a couple of days. This consists of a raw mark only. However, students are invited to the Centre to discuss detailed marks, and strengths and weaknesses, although few do this. The data gathered has been useful to indicate areas of great weakness. The topics of 'Partial fractions' and 'Integration by substitution' particularly, stand out. This has enabled supporting materials to be made available, and some students do make use of them.

The major challenge is not the diagnostic testing itself, but what to do about the results! There are significant numbers of students who are weak across the whole spectrum of basic topics. Referring them to extra remedial material while their main maths course progresses is often the only option available but is rarely seen to work. It takes a particularly highly motivated and mature student to do this. For the majority it is more appropriate to recommend a prolonged period on an alternative course at the right level, but this doesn't seem to happen. Engineering departments want such students to be able to proceed with mainstream engineering courses alongside their well-prepared peers. Major problems are the mismatch between expectations and abilities and a failure to recognise what different levels of mathematical qualification really mean. This results in a tremendous amount of unhappiness and misery for the affected students, and puts teaching staff in a no-win situation.

3.3 Diagnostic Testing at Keele

Dr D Quinney, Department of Mathematics, Keele University

Introduction

There have been considerable changes in traditional student numbers and their qualifications, in addition we have to take into account the increasing number of students who arrive with access and foreign qualifications and the number of mature students. They arrive with a variety of different mathematical knowledge. In order to establish the capabilities of each student it is necessary to identify which skills they are competent in and those for which additional attention is required. Plenary teaching can be complemented by providing supplementary assistance through tutorials, supplementary classes or CAL courseware but this is more effective if each student has received precise and detailed information on their possible deficiencies. The large numbers of students involved means that this is can be time consuming and labour intensive and demonstrates the need for an automated, but relatively simple, diagnostic test or tests.

Diagnostic Test Used at Keele

HoDoMS (Heads of Departments of Mathematical Sciences) has funded a WWW site giving information, contacts and case studies of existing diagnostic tests. Details can be found at:

http:/ /www.keele.ac.uk/depts/ma/diagnostic/

The Mathematics Department at Keele University selected a computer based diagnostic test written at Nottingham University by Hibberd & Brydges. (Brydges S & Hibberd S, 1994, Hibberd 5, 1996.) Customised versions of the test have been implemented by a number of UK universities. The test comprises twenty multiple choice questions selected from a pool of 48 questions to be answered in 40 minutes; selected pseudo-randomly to ensure an adequate range of topic coverage. A typical question is shown below.

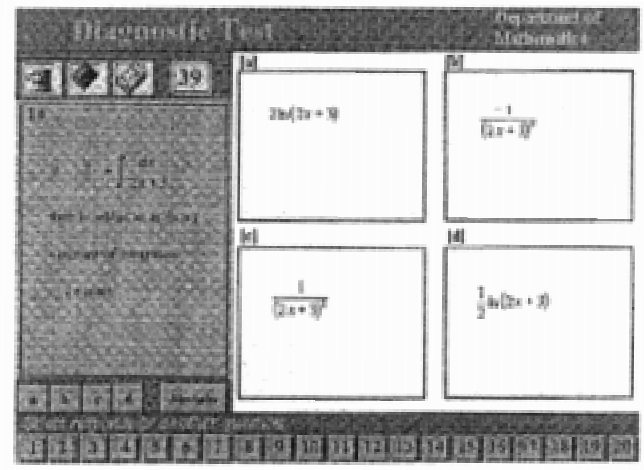
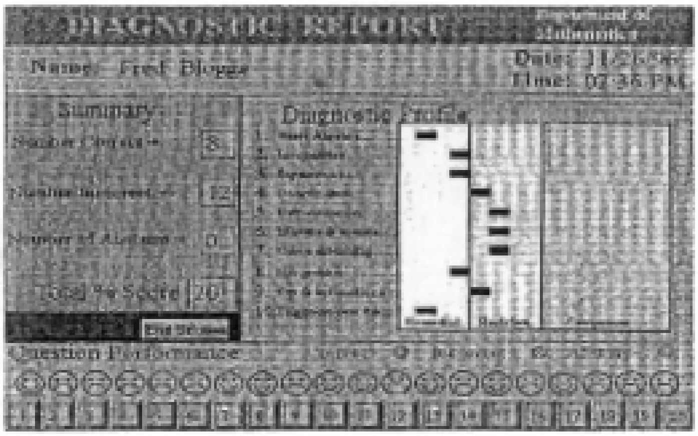


Figure 1: Testing integration.

A total (percentage) score, weighted to minimise ‘guesses’ (+3, for a correct answer and -1 for an incorrect answer), is determined at the end of the test. In addition to a student selecting a correct option they can also abstain; the effect of which is to avoid the penalty of selecting an incorrect answer if unsure. At the end of the test each student is presented with a diagnostic report, see figure 2. Based on this profile it is possible to assign individual support material; in Keele’s case we use Mathwise. Global information on each cohort and question can be readily extracted, including the most common incorrect answers and changes in cohort performance.

Figure 2. Student Diagnostic Report



Although our primary aim is to help students to achieve their potential the gathering of national information regarding the mathematical abilities of students in HE will also be possible. Such information will help long term planning in HE and also provide positive feedback to SCAA and secondary education in general.

Results at Keele University

In addition to providing individual profiles for students it is possible to investigate the skills of the whole cohort. Table 1 shows the average scores of students entering Keele to read mathematics. The scores are normalised so that an "average" student would score 5 in each topic.

Table 1. Keele Results 1996-97

Topic	1996	1997
Basic Algebra	6.62	9.01
Inequalities	7.07	2.25
Expansions	5.67	2.83
Co-ord Geometry	7.06	4.51
Differentiation	3.19	6.34
Max and Min	7.48	5.56
Curve Sketching	6.27	5.25
Integration	3.83	7.24
Log and Exp fns	6.41	8.85
Trig Functions	6.29	0.79

The wide discrepancy indicates that simply selecting all students and providing common courses is not a possible option. Indeed, the variation suggest that a different set of support material would need to be developed each year. To this end, after the student is presented with the profile its results are linked to provide a tailored course which is provided by CAL.

3.4 Diagnostic Testing at Leeds

Professor B Sleeman, Department of Applied Mathematical Studies,
University of Leeds

Diagnostic testing was initially introduced in 1980 with first year mathematics undergraduates. They were given a one hour written test in mechanics to access their basic knowledge and conceptual understanding. For a decade this proved a very useful means of identifying those students who needed extra help/special treatment in order to cope with a demanding first year course in applied mathematics.

In Mathematics

More recently (1998) diagnostic testing in mathematics was introduced following a series of high failure rates, 1ong tails" and a general awareness of a rapid decline in the mathematical knowledge and skills of entrants to mathematics — based degree schemes. There is clearly a gulf between what many students understand/can do and what is expected of them!

A one hour written test was therefore designed — covering algebra, trigonometry, differentiation and integration and given to 450 mathematicians, physicists, engineers and joint honours students. The aims are to

- identify students' mathematical strengths/weaknesses
- identify "students at risk" (and needing streaming)
- assess the significance of the 'A' level Maths grade.

Two rather unexpected results are the following,

(i) students with a good 'A' level Mathematics grade (A or B) cannot be assumed to be competent in basic mathematical skills.

(ii) most students have deficiencies in calculus and display a woeful lack of practice.

With regard to the latter, support is given to physicists by way of a 5 credit Calculus Consolidation module to bring them up to speed.

Appendix 4

Further Evidence of the Decline

Mathematics Diagnostic Tests (1979-1999)

Professor J Mathew, Department of Physics, University of York

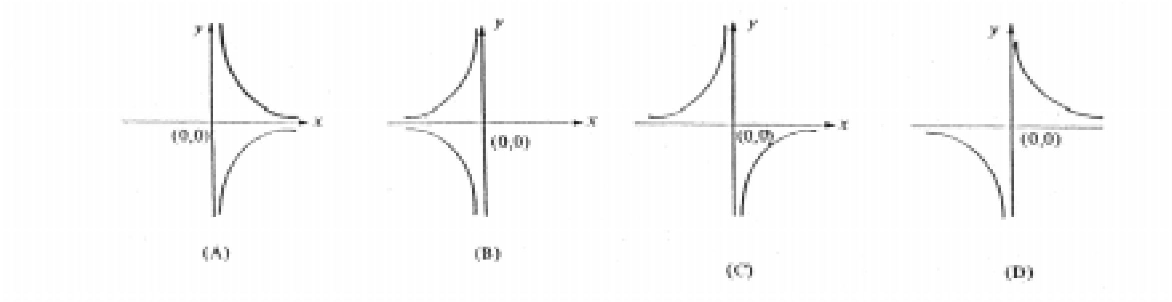
Since 1979 the Physics Department at York has given two multiple choice diagnostic tests, one on Electricity and one on Mathematics, to students entering the course. The tests were developed in York as part of the Physics Interface Project, a joint scheme coordinated at the University of Wales in Cardiff, and based on a Royal Society review of "What Physical Scientists should know when coming to University".

The questions aim to define the level of skill and knowledge retained during the summer after the A-level test. The Mathematics questions reflect both basic pure mathematics and mathematical manipulations relevant to physics, but do not include any Mechanics that might have been covered in Applied Mathematics. There are 50 multiple choice questions with 4-answer choices on each question. Two sample questions are given below:

Q1. $\log_b(2ab) - \log_b(a)$ is,

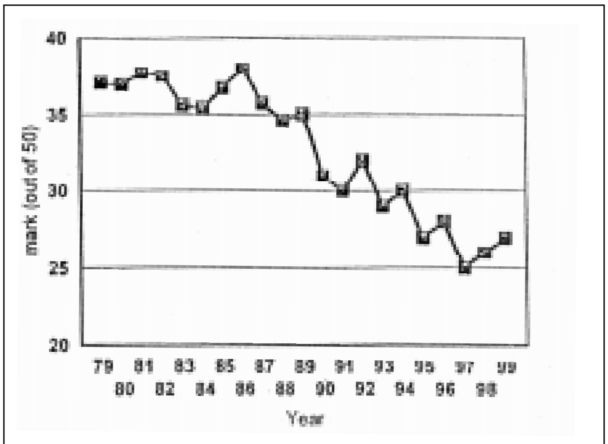
(A) $1 + \log_b 2$ B) $\dfrac{\log_b 2ab}{\log_b a}$ (C) $\log_b 2a^2b$ (D) $\log_b(2ab-b)$

Q2. The graph of the curve represented by the equation $xy = c$, (where $c < 0$) is



Results of the test

The graph below charts the performance of different cohorts from 1979-1999.



Appendix 5

List of Participants

Dr Jack Abramsliky	Qualifications and Curriculum Authority
Dr John Appleby	University of Newcastle upon Tyne
Professor Roger Baker	The Gatsby Charitable Foundation
Professor Keith Ball	University College London
Dr Janice Barton	University of Liverpool
Professor Cliff Beevers	Heriot-Watt University
Dr Pam Bishop	The University of Birmingham
Dr Neil Challis	Sheffield Hallam University
Dr Amanda Chetwynd	Lancaster University
Dr Bill Cox	University of Aston
Dr Tony Croft	Loughborough University
Dr Sue Dunn	Queen Mary Westfield College
Dr Martin Greenhow	Brunel University
Dr David Griffel	University of Bristol
Dr Trevor Hawkes	University of Warwick
Dr Stephen Hibberd	University of Nottingham
Dr Keith Hirst	University of Southampton
Professor Kenneth Houston	University of Ulster
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Dr Duncan Lawson	Coventry University
Dr Sanjoy Mahajan	University of Cambridge
Dr Leslie Mustoe	Loughborough University
Ms Lisa Page	Gatsby Technical Education Project
Dr Judith Perry	University of Cambridge
Mr Roger Porkess	University of Plymouth
Dr Douglas Quinney	Keele University
Professor Chris Robson	Seminar Chairman, University of Leeds
Mr Tom Roper	University of Leeds
Professor Peter Saunders	King’s College London Professor
Professor Mike Savage	University of Leeds
Dr Christine Shiu	Open University
Professor Brian Sleeman	University of Leeds
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Professor Ros Sutherland	University of Bristol
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