

# CETL-MSOR Conference 2007

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## **Papers in MSOR Connections**

The following papers, which were presented at CETL-MSOR 2007, have been published in the May 2008 issue of *MSOR Connections*. These articles can be accessed via [www.mathstore.ac.uk/newsletter](http://www.mathstore.ac.uk/newsletter).

<b>'A' level mathematics and the 3R's – recruitment, retention and reward</b>	<b>P Glaister &amp; E M Glaister</b>
<b>Supporting development through technology and mathematical diversity</b>	<b>H Gretton &amp; M Thomlinson</b>
<b>Overview of the provision of mathematics support to students in a University College</b>	<b>S Parsons</b>
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## Editor's Notes

As was the case last year, reading all the submitted conference papers has been a rewarding experience for me. This time a much larger number of referees was called upon and my thanks go to all of them for their thorough and insightful comments.

An innovation for 2007 has been that a number of papers have been published in *MSOR Connections* rather than in this book of proceedings (see the list on page 7 for the five appearing in the May 2008 issue). This has afforded the double advantage of an earlier publication than otherwise would have happened, and exposure to a much wider audience. My thanks go to all the authors who agreed to that route for dissemination of their work. A consequence is that these proceedings contain rather less papers (18) compared to 2006 (29).

And now to some introductory comments on the 18 published papers ...

### Conference Papers

We were fortunate to have some splendid Key Note addresses at the Conference and two of those are included here. **Prof. Helen MacGillivray** (QUT, Australia) presents a wide-ranging paper on the roles of assessment in the learning of statistics and mathematics, a careful study of which will be of benefit to many readers. A quite different but equally valuable paper by **Prof. Barbara Jaworski** (Loughborough University) maps out approaches to research into mathematics teaching development for the twin goals of enhancing knowledge and improving practice, one of which objectives can often be neglected in the pursuit of the other.

Several papers report research into fundamentally important issues. **Croft, Solomon & Bright** reflect on the undergraduate mathematician's views on learning support, where (perhaps surprisingly) negative views on mathematics are revealed. This is echoed in **Tariq's** paper concerning bioscience students – the paper's title says it all: "Can't do maths, won't do maths – don't want help". Following the same theme, the paper by **Symonds, Lawson & Robinson** reports on an investigation into why struggling students do not take up available support. Also, **Bhakta, Lawson & Goodband** report on what nursing students think about mathematics, and indicate how their very special needs and concerns might best be addressed.

**Golden, Walker & Lumb's** paper on what is required to prepare mathematics graduates for employment might have something to offer in the way of addressing the disillusionment reported by **Croft et al.** Similarly, **Fritz, Peelo, Folkard & Ramirez-Martinell's** paper concerning quantitative skills in the social sciences links with **Bhakta et al's** paper. **Starkings & Maynard's** paper also links with **Bhakta et al's** paper, as they report on two institutions' different approaches to providing support for students on vocational courses (such a nursing), and address the question "Should support be an institution-wide provision or should it be school/department based?"

There were five broadly software-related papers presented at the Conference, all very different. **Currell** reports on using video communication to support a variety of learning styles, which is now becoming a refreshing reality in a number of places, rather than just being a pipe dream. An instance of this being a reality is provided by **Jackson & Tariq's** report on the development of the biosciences equivalent of *mathtutor* which is aptly named *biomathtutor*. The paper by Brunel-based **Hatt & Greenhow** describes issues in developing CAA for graph theory, which provides many challenges. **Lavicza** reports on his large-scale investigation into the use of CAS in HE across three countries. Lastly, in a well-researched and very interesting paper, **Hu & Samuels** provide insight into the use of computer games and mobile technologies to support mathematics learning, primarily through motivation.

**Bidgood, Hunt, Payne & Simonite** report on the outcomes of the PiSA project concerning plagiarism issues in HE statistics. This provides much food for thought and will be of interest right across the (unmanned?) mathematics - statistics border.

Standing apart from all other papers – perhaps appropriate for the topic – **Gordon, Stirling & Swift**'s paper on *Introducing Logic* will be of interest to all who teach mainstream mathematicians.

Mathematics support is often put in the hands of inexperienced postgraduates rather than mainstream academics, and adequate training for such tutors is not always provided. This criticism cannot be levelled at Dublin City University, which has a long history of tutor training, and **Nolan** reports on some further recent innovations using case studies to enhance the tutors' experience.

Mathematics Education is now widely recognised as being a legitimate academic discipline, but what about Mathematics Support? Should it be recognised as a *practical discipline*? **Samuels**' paper considers the case for this. Maybe this is the answer to all our support-related problems– we need more discipline!

The next CETL-MSOR conference will be held at Lancaster University 8-9 September 2008. I look forward keenly to follow-up reports on the disparate research and development work outlined in these proceedings, and to hear of new innovative work for the first time. I hope you will be able to join us, and contribute.

**David Green**



# What nursing students think about mathematics

Roy Bhakta, Duncan Lawson and John Goodband

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## Abstract

It is essential that nursing students have competency in basic mathematics as, in their professional lives, they will be required to carry out drug dosage calculations. The consequences of making errors in such calculations could be fatal. In this paper we will outline information gathered from a series of interviews and focus groups with first year nursing students at Coventry University about their attitudes towards mathematics. Many of the students expressed very negative previous experiences in learning mathematics. This paper will review these attitudes and also differences that emerged between mature students and younger students. Furthermore, this paper will consider some implications for the provision of mathematics support to nursing students and report on a pilot initiative recently undertaken.

## Introduction

Increased variation in the mathematical backgrounds of students can lead to a mismatch between the skills with which students enter university and those that are expected of them [1]. The English education system is different from the majority in the developed world in that mathematical study beyond the age of 16 is not compulsory [2]. Consequently the intakes of many university courses contain a mixture of students; some have, but many have not, studied mathematics in post compulsory education.

Nursing is an example of a course which, whilst not requiring new entrants to have a mathematical background beyond GCSE, does contain a mathematical component. Research suggests that the ability of nursing students at universities in England to perform correctly drug dosage calculations is worryingly low. For example, in one trial involving second year nursing students it was found that only 8% of the students correctly performed 90% of the calculations [3]. None of the students were able to answer correctly all of the questions. It has been identified that there is a need to improve mathematical support in order that nursing students can perform calculations more accurately by the time they graduate [4].

Attitudes can have a significant effect on learning. The ways in which students approach learning mathematics (e.g. instrumental learning or relational learning [5]), and their performance in mathematical tasks, have been found to be strongly influenced by their personal attitudes towards mathematics [6-9]. Positive attitudes towards the benefits of studying may be greater for students who believe intelligence is a result of prolonged study rather than being genetically predetermined [10]. Moreover, maths anxiety can have a significant and measurable negative effect on performance [11].

This paper describes preliminary research carried out at Coventry University investigating the attitudes of nursing students towards mathematics and their preferred ways of learning.

## Method

Coventry University has approximately 250 first year Nursing and Midwifery students who range in age from 18 to 50; the majority of them are female. All are required to have, on entry, a minimum of a GCSE grade C in mathematics (or equivalent) or to take a university mathematics test. Ten of these students (all female) participated in this study; three were over 21 years of age ('mature') and the other 7 were under 21 years of age ('ordinary'). The mature students tended to have returned to education from a working background or from looking after a family. These students typically have not studied mathematics for a number of years prior to entering onto the course. Ordinary students have not been out of education for any significant amount of time. Typically the ordinary students have moved from school to college and now to university. None of the 10 students had studied mathematics beyond GCSE.

The selection of students for this study was opportunistic. Nursing and Midwifery students involved in a sigma (see Endnote 1) run workshop were asked to volunteer to participate in the study. Whilst the ten students who participated cannot be viewed as a random sample of students from the course, the qualitative information gathered from them can be useful in shedding light on some of the attitudes towards mathematics held by nursing students.

Data were obtained through a combination of individual interviews and focus groups during the second term of study. Focus groups and interviews were aimed at eliciting students' own perceptions of their experiences and attitudes towards mathematics. Participants were able to choose the composition of the focus groups in which they participated. A maximum group size of three was permitted with the option available for participating alone. The groups tended to consist of students who were friends.

## Results and findings

Interview data suggested that many of the students' feelings towards mathematics and their current preferences for learning may be associated with a number of factors from their past learning experiences. The next four subsections give an account of the four main themes raised during the interviews. It must be emphasised that the accounts are based on the students' perceptions of past events which may differ from the actual events. It has also been assumed that these perceptions have been accurately communicated in such a way that the language and usage of words has the same meaning for both interviewer and participant.

### Past experiences and attitudes

Students expressed a mixture of both positive and negative feelings towards mathematics; however it was interesting to note that the majority of the negative feelings were centred around their experiences at secondary school. These negative experiences were linked to the types of learning experiences the students encountered whilst at secondary school which the students described as being significantly different from primary school where mathematics was viewed as being fun, playful and game like.

Past experiences and the feelings associated with them were linked to the school/educational environment and not strictly with the idea of mathematics in general. Much of the negativity towards mathematics seemed to be associated with the school environment, and related to feelings of stress, pressure and anxiety as a result of the change in teaching and learning techniques (more board work, less teacher contact time, working individually from text books) or through the perceived increase in testing and exams.

An ordinary student commented:

*"... they started to more or less sort of put books into the equation, like copying things from a book ....*

*... the time with the teacher was really different. I think that's when I most started to struggle really and realised that you can't have someone working with you on a one to one...*

*...it weren't as [much] fun ... the support is not there as much."*

In general the students felt that the fun had been removed from mathematics when entering secondary school and as a result their interest in the subject had waned. Alongside the negative feelings about the educational environment were negative feelings associated with the content of school mathematics which was perceived to be irrelevant. This also had an impact on the motivation to engage with the material learnt. All of the students felt that there were two kinds of mathematics: the relevant and the irrelevant. There was a general feeling that secondary school mathematics was of little relevance to their normal day to day functioning. An ordinary student commented :

*"...at secondary school the trigonometry and all that kind of sort of thing, I've never used it...and it just seems like something they used to stress us out ...since then I've never been able to kind of think of a situation where I've applied it."*

However all of the students, particularly the more mature students who had had a greater exposure to work and social environments, agreed that mathematics is important and that, at present, their competency for dealing with day to day mathematics was adequate. It is interesting to note that at no point did the students (ordinary or mature) express any negative feelings regarding mathematics when discussing its use outside of the educational environment (e.g. accounting, dealing with money, currency conversion, time calculations).

Students' reasons for actually participating in the learning of mathematics also changed. During early childhood the students described their motivations as being predominantly out of 'fun' or the desire to win at some kind of game or play activity. During secondary school it was reported that they felt no internal motivation to partake in the activities and learning. Motivation had an external source, mainly the drive of the teacher who wanted the student to learn or to complete exercises. Post 16 (key skills, BTEC, GNVQ) and in the workplace (on the job training) the source of motivation reverted back to the students who realised that mathematics was necessary for their chosen career.

Interestingly, the mature students were more able to suppress their negative feelings as for them there was often greater pressure to succeed (for example, getting a well paid job to support a family). Such students were more willing to seek out support where available.

### **Working in groups**

All the students preferred to work in groups whilst studying mathematics rather than working in isolation. They felt this was a more effective way of learning. An ordinary student commented:

*"I do like to work in a group. I find it good to work in a group because then you get different perspectives."*

The composition of the groups was important to students and opinions varied; the majority preferred similar rather than mixed ability groups. Some students identified issues related to both types of groups. With similar ability groups there was a chance that all may encounter the same problem, meaning that the whole group cannot make progress. Mixed ability groups were perceived as not having the same problem. However it was thought that weaker students may find themselves lacking confidence due to the stronger students dominating the group and as a consequence they may not make full use of the opportunity for learning.

In a group study environment where students could be working on the same or similar problems, the majority of students felt that working in groups with students from more mathematical disciplines would be immensely distressing. However they did indicate that intervention from a member of staff or a more able member of the group acting as a 'teacher' was welcomed.

### **Interactive and fun activities**

Many of the positive feelings towards mathematics date back to pre-secondary school experiences where learning mathematics was tactile and playful in nature. It was felt not to be a chore but an enjoyable activity to take part in. Mathematics at secondary school was perceived to be dull and boring as the learning methods changed with book and board work replacing interactive activities. Two ordinary students said:

*"I remember it being more... instead of written, being pictures and more graphs which obviously made it a bit more enjoyable"*

*"You had to colour in 5 or something like that"*

*"It was really quite simple maths, but enjoyable"*

*"Activities were more interactive rather than just sitting down"*

Students felt that activities which are tactile (for example, using physical objects like syringes and drips) can help their current learning in mathematics such as improving skills of estimation and the ability to appreciate quantities in different units.

### **Visualisation and procedural strategies**

Some comments were also made on how students individually learn mathematics. Descriptions tended to focus firstly on visualisation where students attempt to tackle mathematics problems through the recall of environments where they originally learnt the mathematics. In some cases the students went so far as to see themselves in a mental representation of their classroom and even recalled the pages of a text or exercise book which was used at the time. Several students also touched on the idea of performing calculations mentally by creating mental images of physical objects and then performing operations on these objects so as to solve the problem (e.g. mentally picturing pills of varying doses and then grouping/sorting/adding them). Drug calculations were thus being performed through the manipulation of mental representations of physical objects rather than symbolically. Secondly, some of the students found that solving mathematics problems for them involved the need to learn a multitude of rules rather than moving towards a more abstract understanding of the subject matter:

*"I try and remember the little golden rules like there's 100cm in a metre... try and think of all those first before I try and tackle the question."*

### **Discussion**

Data gathered from the interviews suggest that students have specific preferences for learning and that their attitudes towards mathematics and effective learning are strongly influenced by their previous experiences of mathematics in both the working and educational environments.

Students perceived three kinds of mathematics. The first was associated with work and social environments where it was seen as a tool or a skill that could be utilised. The second was pleasurable interactive activities experienced in primary school mathematics. The third was mathematics taught in the formal environment of secondary school where it was associated with a body of knowledge that had to be learnt to pass exams but was perceived to be of little use to the student outside the classroom. From the interviews it would appear that many students still associate formal mathematics teaching with feelings (for example, mathematics is irrelevant or boring) originating from their secondary education.

Consideration of the data gathered in this study suggests a number of ways in which it may be possible to improve the learning experience of nursing students.

1. There appears to be a strong preference for working in groups. This practice should be facilitated through the provision of appropriate study facilities.
2. As far as possible a learning environment should be provided which is free of the factors that produced negative feelings towards mathematics in secondary education.
3. The contextual relevance of mathematics to their workplace should be reinforced through the demonstration of calculations in a simulated work situation using real equipment. However, the demonstrations should seek to move the students beyond relying on procedural solutions to a conceptual understanding of the mathematics.

### **Focused support**

Comments made during the interviews indicate that nursing students were easily intimidated by the presence of students of other (more numerate) disciplines whom they regarded as mathematically very able. These feelings may explain the comparatively low usage by nursing students of the Mathematics Support Centre at Coventry. In order to address these feelings two series of drop-in workshops, in March and May, were provided solely for nursing students. The mathematics covered in these workshops was highly contextualised, relating specifically to the kinds of calculations nurses have to make in professional practice (such as unit conversions, drug dosages and drip rates).

The first series of workshops were held in a seminar room in the nursing department and the second series were held in a room close to (but completely separate from) the Mathematics Support Centre. Each series of workshops took place over four weeks; 31 students made a total of 40 visits in the first series and 39 students made a total of 44 visits during the second series. This compares favourably to only 8 students making a total of 11 visits to the Mathematics Support Centre during the first five months of the academic year.

It was noticeable that the overwhelming majority of visits to the workshop were made by students in small groups (usually two or three students at a time). There was a strong feeling of peer support within these groups, with a sense of fellow students enabling those lacking confidence to overcome their feelings of anxiety or inferiority. Following the second series of workshops, the nursing course tutor provided the following feedback: "I know from feedback that the students found your input very useful. Students informed each other of the workshops and were eager to let each other know about them."

### **Conclusion**

Many nursing students need support in order to cope with the mathematical elements of their course. However, due to negative experiences related to learning mathematics in secondary school, they often view mathematics as irrelevant and, in addition, lack confidence in their mathematical abilities. As a consequence, they are reluctant to take advantage of generic mathematics support. They are more likely to engage with directly focused support which contextualises the mathematics to make it relevant to future professional practice and which provides them with opportunity to work in small groups that provide peer support and mutual encouragement.

### **Endnote 1**

**sigma** is a HEFCE funded Centre for Excellence in Teaching and Learning (CETL) in Mathematics and Statistics Support. **sigma**'s primary function is to provide support to students across the University who require mathematical or statistical skills as part of their programme of study.

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# The PiSA (Plagiarism in Statistics Assessment) Project

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## Abstract

There is much concern in British Higher Education Institutions that instances of plagiarism are on the increase. Forms of assessment such as the traditional examination and on-line testing using large question banks and randomly created tests are the least vulnerable to academic misconduct, but these types of testing fail to address some important learning outcomes in Statistics, not least the ability of students to analyse a set of data appropriately and report results effectively without very limiting time constraints. However, the lack of supervision associated with coursework assignments means that giving students the same data to analyse has serious risks of plagiarism, either in the analysis, or in the reporting. Group work, which is used to give students opportunities to develop team skills, has its own plagiarism problems. This paper reports on the PiSA (Plagiarism in Statistics Assessment) project which surveyed Statistics lecturers in Higher Education in order to identify areas of good practice in methods of assessment and strategies to deter plagiarism that are being used. These include giving each student a unique random sample from a larger dataset or developing methodologies to allocate marks fairly in group work.

## Introduction

Plagiarism "To take and use as one's own the thoughts, writings or inventions of another" (Oxford English Dictionary) is thought to be increasing within British Higher Education (HE) institutions. Traditional examinations and tests, even if they are on-line, using large question banks, are rarely vulnerable to plagiarism, as they are usually carefully supervised. "Take-home" coursework, whether group or individual assignments, and final year projects are the forms of assessment most susceptible to plagiarism in the forms of copying or collusion. However, giving students opportunities to analyse a set of data appropriately and report results effectively without very limiting time constraints addresses some central learning outcomes in Statistics. This sort of activity is important in both service courses, where typically students will perform some exploratory data analysis, and in more advanced modules, where statistical modelling might be appropriate.

During 2006-2007, the Royal Statistical Society Centre for Statistical Education (RSSCSE) and the Mathematics, Statistics and Operational Research (MSOR) Network jointly funded the Plagiarism in Statistics Assessment (PiSA) project, which aimed to:

- survey HE lecturers in Statistics to find out what methods of assessment and strategies to deter plagiarism are being employed currently;
- identify and synthesise elements of good practice;
- disseminate findings widely.

The focus was on: whether Statistics lecturers thought that coursework was important; whether they were alert to copying and collusion; how they were tackling these issues, including whether they had reduced the amount of coursework because of concerns about plagiarism. Given this information, the project team would identify and disseminate areas of good practice.

## **Evidence gathering**

Evidence was gathered from a number of sources – responses to an article in MSOR Connections [1] ; a JISCmail discussion list; a website; an email request to HE Academy Subject Centres that used Statistics heavily and a survey of lecturers in 23 UK universities.

The MSOR Connections article outlined the project and asked anyone who had

*“... some good practice in plagiarism detection or deterrence that you would like to share and/or if you would be willing to discuss your concerns about the effect plagiarism is having in Statistics assessments...”*

to contact a member of the project team. Further, the article gave the address of the JISCmail discussion list [2]. The responses here were on three main themes. Firstly, the extent to which students should be able to discuss their work, with the consensus being that this was to be encouraged, although there were then possible problems in assessing individual work. Secondly, whether individualising assignments by students collecting or simulating their own data was worthwhile, and finally, the design of plagiarism-resistant assessments.

The website, established under the MSOR Network address, gave a facility for both staff and students to give their views, each section with some relevant questions “to spark your thoughts”. For the lecturers these were about noticing an increase in cheating, what strategies were used to combat plagiarism and whether this had meant changes in how students were assessed. For students the questions focussed on types of cheating, whether fellow students got caught and what the punishment should be. The lecturers comments were on how they had tried to tackle plagiarism; although there were only 2 student respondents, each highlighted an important aspect – one was concerned that group work was not always marked fairly as some members did not contribute as much as others, whilst the other brought attention to cheating companies where students could pay someone else to do their work.

An e-mail request was sent out to lecturers who use Statistics in other subjects through the HE Academy Subject Centres in Bioscience, Business, Economics, Geography, Health and Psychology. There were a number of replies here, focussing on procedures which helped overcome plagiarism, particularly issues concerning large classes, which are common in these discipline areas.

The survey of lecturers was by personal interview by ‘phone, email or visit with one of the project team consulting willing volunteers. Thus there was no attempt at a random survey of the whole of HE; as the focus was not on prevalence, nor case history, but rather good practice, this should not matter. However, this survey elicited responses from 50 lecturers at 23 different universities who commented on 96 taught modules as well as on final year projects. This review covered both “old” and “new” universities and service as well as main stream Statistics modules. The majority (53) of the taught modules and all the projects were part of Mathematics or Statistics degrees, 26 replies concerned modules on Business or Economics, 10 from Health or Biosciences, 6 from Psychology and 1 from Computing.

## **Plagiarism deterrence**

One respondent made the point that there are three types of assessment commonly used in Statistics – “mathematical” where there is relatively little scope for individual or alternative approaches to problems,

“practical/analytical” in which students are required to analyse data and write a report, so there is more opportunity for individual creativity, and “computing” tasks. Unsurprisingly, most comments and suggestions on deterrence were made on the latter two types.

Several ways in which plagiarism was deterred were identified from the evidence gathered – through institutional procedures, organisational measures, supervised assessments, individualised assessments, student-centred assessments and electronic submission.

Publication of institutional procedures can be valuable in deterring plagiarism. Typically, students are informed about what is and what is not acceptable; several lecturers reported that they gave students case histories of previous plagiarism prosecutions in Statistics. In many universities students have to declare, in writing, that the work they submit is their own, which at least makes pleas of ignorance untenable. However, some lecturers dealt with “minor” cases of copying or collusion informally, warning students that future misconduct would be dealt with more severely; this is partly in response to over-bureaucratic university procedures, but has problems that students may complain that they are not treated fairly if different lecturers behave in different ways.

Within organisational measures, marking is one of the main issues, where there is much difference between specialist Statistics modules which are typically small and very large service modules. In the former, lecturers generally did not see plagiarism as an issue, since they knew their students well, marked all the work themselves and so believed that they would be able to detect copying or collusion easily. However, with large classes, where it is usual to have many markers for each assignment, copying could easily go undetected. Some ways suggested to avoid this are to look closely at students’ work where there is a large discrepancy between coursework and examination marks; allocate markers so that each assesses one part of all students’ work; distribute marking at random, so avoiding copying from a different tutorial group; double mark a random selection of assignments.

Many lecturers used some form of name, ID number or time stamp within submitted computerised assignments, so that students were deterred from handing in a doctored copy of another’s work – this could be a name in the headers and footers of each page of a word-processed report, time stamping of plots within software or name and ID as a caption on each graph.

Group work, which gives students opportunities to develop team skills, has its own plagiarism problems, when there are members who do not contribute and hence can gain credit for others’ work. It is often up to the group itself to identify non-contributors and report them, so that they receive fewer or perhaps no marks. One solution is to allocate students to groups of 2 or 3 as a deterrent to the non-contribution of some members. Some lecturers use presentations, including posters, as a method of asking members in turn about aspects of the work, so that they would need some understanding of the analysis in order to answer convincingly. In other situations, students, although they have collaborated within a group, are set individual, time constrained tests on their findings to assess their understanding.

One response to plagiarism problems has been a move from “take-home” assignments towards supervised assessments. In order to fulfil some of the learning outcomes, rather than this just being a standard test, typically, each student is given some data set, case study or research paper in advance, which they can discuss with others but crucially, the final assessment is an individual piece of work. This, however, has its own problems and staff reported cases where rooms were unsuitable, for example conducting a test in a lecture theatre; the difficulty in checking student identities when there were an inadequate number of invigilators, and possible failure of equipment when the assessments were in computer laboratories. There were also problems with large classes where students often sat tests at different times so cross-class collusion was possible.

Many lecturers have developed ways of individualising assessment tasks using technical, software dependent methods. For example, using the assessment tool within the university’s virtual learning environment to create randomised computer-marked online quizzes was used by several lecturers as a pragmatic response to assessing large classes.

There were several variations on using a student's ID number to allocate individual subsets of a larger dataset. For example, ISCUS (Individualised Statistics Coursework Using Spreadsheets) developed at Coventry University enables a lecturer to create an assignment generator spreadsheet into which students enter their ID number to obtain data and/or tasks [3]. DRUID (Dynamic Resources Using Interesting Data) from Nottingham Trent University allows a unique worksheet, dataset and solution sheet for each member of the class [4]. Other examples of using a student's ID, but in a simpler way, were to take the last two digits of the number, say X and Y, and either to ask the student to calculate a 9X% confidence interval, or to exclude rows X and Y from a given dataset prior to analysis; also, by an online simulator to produce datasets based on the student's ID number or designing the questions so that solutions are all numeric or multiple-choice, which aids marking when students have different data to analyse.

Many assessment tasks require students to collect or simulate their own data. In order to deter plagiarism, lecturers have used a number of strategies – for example data on the same topic, but each student has to get information from a different country, or from a different journal. Alternatively students collect data on different topics, but they have to be the most recent available, thus reducing possibilities of copying across years. However, there are risks here as students may choose data from the internet or a textbook for which the analysis is also available. In order to avoid this, the source of the data needs to be specified well before submission date so that lecturers can check its suitability.

Although most responses were about taught modules, many universities require (or at least encourage) their students to complete a project in their final year. Typically, these are individual pieces of work that are considered to be the equivalent of one or two taught modules. Given this weighting, the potential for plagiarism needs to be taken very seriously. At many universities, some of the marks are given for “development” or “ability to progress the project along appropriate lines” or “time management”. Thus students need to meet regularly with their project supervisors to discuss progress and to work consistently if they are to gain these marks. Also, in many universities, students are given a final viva by at least two members of staff, which has proved a useful way to detect plagiarism.

Requiring students to submit their work electronically can help with checking for plagiarism. File properties, such as author, location, and times of creation, modification and access, can be scrutinised as a method of plagiarism checking, although some students may well find ways around this. A more sophisticated plagiarism detection tool is TURNITIN, which is recommended by JISC and now available in almost 90% of UK universities. TURNITIN checks for intra-class and inter-year collusion as well as internet plagiarism. Some lecturers felt that their confidence in the students' coursework was increased using this tool, or just with students knowing that it could be used. However, TURNITIN needs to be used with some caution, as students are encouraged to use standard forms of technical language when reporting their findings and this can often lead to false instances of copying being reported. In this case, lecturers need to set a threshold percentage similarity below which they will not investigate for plagiarism.

Lecturers also need to be aware that, whatever procedures they put in place, there is an increasing number of cheating companies that are prepared to provide coursework and projects for various prices. Typically these companies employ a panel of experts who effectively act as consultants to students who employ them. Detection arises when students submit coursework in better English than that which they usually use or their coursework marks are much higher than test or examination marks.

## Conclusions

All lecturers need to give serious attention to detecting plagiarism and develop strategies to deter it. The majority of Statistics lecturers are well aware of plagiarism issues and are taking some action, however limited, to combat it.

Institutional procedures against plagiarism need to be clear, consistently applied, with a graduated scale of penalties and involve minimum bureaucracy if they are to be used by staff and respected by students.

Plagiarism is harder to detect in large service modules, where organisation of marking is an important issue, not least in finding ways to deter copying or collusion. Small specialist Statistics groups are not as prone to this as the students are well known by their lecturers.

There is a great deal of innovative work in the area of individualising assessment tasks, much of a technical nature, based on using the students' IDs. Assessments that require students to collect their own data are widely employed.

There has been a move away from take-home assignments to in-class, supervised tests. These generally encourage students to discuss the data and analysis in small groups beforehand, although the final assessment is an individual piece of work. However, in-class tests can be exposed to cheating themselves where there is unsuitable accommodation or inadequate invigilation.

Where assignments are submitted electronically TURNITIN, for example, can be used to check for plagiarism and collusion. This is particularly useful for final year projects and can also help to detect plagiarism within assessment solutions bought from one of the on-line cheating companies.

Despite all the problems with plagiarism, there is a consensus that coursework assignments where students have to analyse data and report on their findings is an important part in any Statistics module, reflecting the applied nature of the discipline. Lecturers need to be vigilant to instances of copying and collusion and many are combating this with innovative measures.

A full report of the findings from the PiSA project, including many instances of good practice, can be found on the website [5].

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## Developing academic support for mathematics undergraduates – the students' views

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### Abstract

The last decade or so has seen efforts made throughout the higher education sector to provide support to students *embarking* upon courses with a significant mathematics requirement. Recently, findings from several research projects report disillusionment and disengagement amongst specialists *at year two and beyond*. Evidence from several recent reviews raises concerns about the quality and quantity of UK PhD entrants in mathematics. This paper reports research intended to explore improvements to the support of specialist mathematicians in order that more of them retain a positive attitude to studying mathematics, and those who want to undertake postgraduate work are better prepared. The paper focuses on second year undergraduates, their opinions and experiences of mathematics at university, and the support they call upon. Focus groups were conducted in two universities yielding interesting perspectives on peer support, independent learning and how students see their careers developing.

### Background and motivation

The mathematics Higher Education (HE) community is well aware of the challenges associated with teaching at the *school/university interface*. Problems in terms of skills deficits have been reported regularly [e.g. 1]. The 'mathematics problem' has several other dimensions – ones which exist beyond the interface and which impact upon the specialist mathematics community. Several researchers have observed that undergraduate mathematicians are not well-equipped to deal with proof [e.g. 2...5]. Undergraduates "*exhibit a lack of concern for meaning, a lack of appreciation of proof as a functional tool and an inadequate epistemology*" [6]. Crawford et al. [7] note the related effect that first year mathematics students see mathematics learning simply as a rote learning task, while Hoyles et al.'s [8] close study of changing patterns of transition from school to university uncovers a number of mismatches between A-level and university mathematics: "*The fact that school mathematics is so different from university mathematics, and that tutors recognise that there is a jump, implies that the change in content at A-level may play less of a role than we think in determining the facility with which students make the transition*". Solomon [9] similarly has queried whether difficulties with proof are due to a skills deficit or, rather, to a less obvious gap in values, expectations and beliefs about mathematics itself which are exacerbated by the recent market trends in HE. This same study found little evidence of the identity shift towards a more autonomous and participatory role that one might expect or hope for as a result of a university learning environment. Solomon [10] reports on a pervasive identity of 'not belonging'. Understanding the developing experiences of undergraduate mathematicians is, therefore, of importance.

A major contribution in this area is Brown et al.'s ESRC-funded project, *Student Experiences of Undergraduate Mathematics* (SEUM), which examined single honours mathematics undergraduates in two research-led universities [11]. Their 2002 report notes that "*for many of those staying [on the course] attainment was average*

and below, the problems of coping with the work were accompanied by growing disillusionment with mathematics; generally, although with some exceptions, students' enjoyment of the subject declined over time". Many did not adapt well to develop new styles of working: "Such students became mildly depressed in the second year and seemed to lack immediate sources of support and the motivation to seek these out". From the same study, Macrae et al. [12] write: "it is difficult to know what more the university could do to support these struggling students... However, faced with widening participation, universities need to put in place increased support structures to encourage struggling [second year undergraduate] students to seek help before it is too late".

Beyond the second year of study there is, not surprisingly, evidence of further problems. Concerns have been expressed about the quality and numbers of UK PhD entrants in the mathematical sciences and cognate disciplines. The Roberts review [13] draws attention to the quality of PhD entrants to Science, Engineering and Technology departments: "A particular concern of many respondents to the Review was the quality of PhD students, both at the commencement of their study and on completion of it" although the review did note that no firm conclusion should be drawn from their data in respect of mathematics. The report [14], notes: "the domestic supply of mathematically competent manpower is in such decline that in many areas we are now dependent on trawling recruits from other countries" and "It becomes essential to ensure that our national curriculum and incentive structure allows our schools and universities to produce home-grown research mathematicians of sufficient calibre to compete with those from other countries." Commenting upon the adequacy of the current three year PhD model prevalent in the UK the review [15] noted: "The system of three-year PhDs can only work if there is excellent A level education at the school level. Our perception is that A levels are weaker than they used to be. The result then is that this produces many students who cannot compete with graduates from abroad".

## Research methodology and samples

The background described above has motivated the current project which seeks to learn more about undergraduate mathematicians, their aspirations and especially their views on the support they think beneficial. We set about this project because we were interested in ways in which support for undergraduate mathematicians might be enhanced. Moreover, the research of Brown et al. [11] indicates that there may be scope for improvement to support beyond year one. This is true not just for struggling students but also for those who are amongst the best, who might be better supported to become even better mathematicians, some of whom may be encouraged to embark upon research careers in mathematics. So, we are focussing efforts on the support of students who have successfully completed year one courses. We have adopted an *Action Research* methodology embarking upon several cycles of activity during which we will implement strategies for improvement, evaluate and refine them, and repeat over at least two years. To explore the issues more fully and underpin practical developments within a theoretical framework we have drawn on existing research on the social practices of undergraduate mathematicians in addition to the data gathered during the course of the project to inform new initiatives. We have been interested to hear what undergraduates have to say about their experiences and expectations of university mathematics, and how their experiences change as they progress. In particular we are interested in their academic support needs and practices – which forms of support they access, which forms of provision they find inaccessible, their views on support provided formally by the institution, on less formal contacts with academic staff and postgraduate tutors, and on the informal networks they have built up with their peers. We were also interested in their future aspirations and whether these had changed whilst at university.

Our approach has been to conduct focus groups with second year undergraduate mathematicians in two research-led universities, A and B. The first set of focus groups were conducted in University A in November 2006. Undergraduate single honours mathematicians who had performed well in year one, obtaining upper-second class honours or above, were invited to participate as these could, potentially, be students with the ability to undertake postgraduate work. Ten students volunteered, enabling us to conduct two focus groups of five

participants each. The second set of focus groups were conducted at University B in May 2007. This time, all the students on the single honours mathematics course were invited to participate. Several volunteered, but only five attended, so two focus groups of two and three participants were organised. The focus groups were conducted in a non-threatening social environment, and guided by questions that had been predetermined by the facilitator. These were generally open-ended to foster discussion amongst the participants.

## The theoretical framework

Several researchers have observed that even within the undergraduate mathematics community there are some students who hold negative views of learning mathematics, and there are those who despite being well-qualified are insecure in their own abilities or lack confidence [e.g. 10,11]. In order to understand how these students develop negative relationships with mathematics, it is useful to explore their identities in terms of their membership of a community of practice [16] and the extent to which they experience studying mathematics in terms of *“legitimate peripheral participation”* [17] – that is, as a novice who feels guided and supported by experts. This socio-cultural perspective characterises identity as the experience of a common enterprise, with shared values, assumptions, purpose and rules of engagement and communication: *“we know who we are by what is familiar, understandable, usable, negotiable; we know who we are not by what is foreign, opaque, unwieldy, unproductive”* [16]. The research literature reports that many students in fact tend to describe themselves as outsiders, as lacking control over their mathematical knowledge and its learning; they follow rules without understanding, and consequently they are vulnerable to failure – staying with the subject is possible only as long as they can do it, and this ability can fail at any time. Brown & Macrae [18] report that many students choose to study mathematics at university because they find it easy at A-level. Thus they risk losing motivation when the work becomes more difficult and success is no longer guaranteed.

The SEUM project also reports more generally on students’ emotional responses to their studies, finding that the certainty associated with mathematical knowledge transfers to a sense of security and pleasure when doing mathematics and producing the solution, but that it can be emotionally demanding when it is a struggle. Of particular relevance here is Brown & Macrae’s [18] finding that students who had more positive attitudes to studying mathematics were those who shared their ideas and problems with other students. Feeling part of a mathematical community emerged as a crucial factor in the student experience, and in the SEUM project, this community focused on a particular physical space within one of the participating universities.

## Analysis

The first stage in the analysis of the focus group data was to identify key themes arising from the discussion and to locate student responses within these themes. The emergent themes were categorised as: views of mathematics at university, transitional issues, academic support, and the students’ futures.

**Mathematics at university:** students talked about their views of participation and interaction, the influence of lectures, approaches to new topics. Students’ opinions of tutorials including factors like group size, attendance, effectiveness, were of interest. Students’ perceptions of lecturing staff and their roles, and students’ relationships with their peers also emerged.

**Transitional issues:** a particular area of discussion was the transition from year one to year two. Factors such as changes in workload, difficulty, assessment, and independence were highlighted. Where the students perceived an increase in workload we were interested in how they intended coping with this.

**Academic support:** students described their preferred sources of academic support, both formal and informal, and whether the different forms of support were accessible and useful.

**Their futures:** students talked about what they thought was still to come whilst they were at university, and then what their careers might look like beyond. They spoke about their awareness of the opportunities open to them. We would be interested in whether their university experience had changed their aspirations and in what ways.

A “long-table” approach [19] was adopted to locate responses within these themes. Through the focus groups a considerable amount of qualitative data was generated. There is space here to describe and discuss just some of the findings which emerged.

**Mathematics at university:** All students in the University A focus groups were positive about their university experience. On the other hand, when discussing mathematics at university, students at University B made the point that they don't feel like part of the mathematics community. Unlike at school, there are so many students, and they are not known by the teaching staff.

*“Here you're just kind of a number, or one of a crowd. You feel quite anonymous and you've got all this learning to do.”*

**Transitional issues:** Much of the discussion related to the transition from year one to year two. Students in the University A focus groups found that, at the time the groups were held, there was not a great deal of difference in workload between year one and year two. However, they preferred the assessment structure in year one referring explicitly to a greater emphasis in year one on coursework. On the other hand, students from University B described how they found it harder to understand new material in year two. One student mentioned they had much less supervision, tutorials were different in terms of group size and lecturer, and so felt there to be less support.

*“This is the first year I've gone into an exam and had questions I can't actually answer, because I've always been able to do all of the paper. There's a lot of material this time round, but unless you understand it, it's not really interesting really.”*

*“Now I do have to work hard, I don't wanna do it. I'm just lethargic because I've not done it before in my life and it's even worse because it's second year I have a sort of don't care sort of attitude because I've been able to fly through things in the past.”*

At University A the biggest transitional change concerned independence. When asked about ways in which their learning was different, several students referred to the need to be more independent:

*“this year its a lot more independent, and you've got nothing to force you”* [a reference to a decreased emphasis on assessment through coursework]

and some recognised that they were not necessarily very good at this:

*“I'm not very good at independent studying – I'm not very motivated”*

**Academic Support:** In the first focus group one student commented that he would access support in the first instance from his personal tutor. However once a second student mentioned that he would seek help from his mathematician flatmates, this changed the responses from others in the group. It became apparent to the students that, in fact, they were much more likely to seek help from their peers:

*“I forgot about that actually - that's probably actually what I'd do before I'd see my tutor.”*

The second focus group made reference to a member of the class, student S, (whom the others declared a *genius*):

*“first response is to go to S!”*

Further discussion revealed that whilst the students did not at first explicitly recognise this, the help and support they were obtaining from their peers was in fact highly valued. To clarify we asked, at the end of session, whether this was the case:

*“definitely”, “the most common I think, maybe not always the best way”, “yeah I reckon my friends will be the first point of call”*

When the question was asked ‘where do you go for your first point of support?’ to students in University B, the unanimous answer was “friends.”

*“that’s the most fruitful situation in which I work – and sit with a group of friends”*

Students at University B also made reference to textbooks, but feelings were mixed as is evident from these responses from two students:

*“I’ve only bought one book, but that was a mistake”*

*“if I don’t understand something, the first thing is to go and buy a book”*

Futures: In terms of their futures, no students from University A were interested in postgraduate mathematics. A disturbing finding from all University B’s five students related to the fact that all were very good at mathematics at school and that they had wanted to become ‘mathematicians’. Their university experience, rather than nurturing this aspiration, had had the opposite effect.

*“The idea was to become some sort of maths guru, but it just doesn’t happen, I just don’t understand it.”*

*“This year’s put me off a bit, it’s just an effort to do maths now, it’s not enjoyable really.”*

*“When I first came to uni, I had high hopes and wanted to do maths as a PhD and now I don’t even want to do maths, I want my education to be over as soon as possible.”*

## Discussion

Our findings support those cited earlier - even amongst well-qualified second year undergraduate mathematicians there are those with negative views of learning mathematics. Our hypothesis is that these negative views might be ameliorated through actions aimed at improving the extent to which students can become more engaged in both peer communities and communities involving both novices and experts. A key finding from our focus groups has been that many students value, first and foremost, the support of their peers. A second finding has been that students need more explicit help to become independent learners. In terms of our Action Research project, our Action now is to put in place measures to facilitate and encourage opportunities for peer support, and provide activities intended to develop independent learning skills. Throughout 2007-8 we are implementing these actions, which will be reported upon at subsequent conferences. Clearly through our Action Research project we will be able to develop explicitly additional support for students at our own university. We seek to influence improvements in support elsewhere through dissemination of findings and professional development activities organised through sigma CETL and the MSOR Network.

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## Video Communication to support variety in learning styles

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### Abstract

This paper reports on the effectiveness of developments that were proposed at the beginning of the 2006/2007 academic year in relation to modules that teach mathematics and statistics to approximately 300 first year science students at the University of the West of England. The very successful introduction of screen-capture video clips together with a student-led focus in the organisation of the modules have produced a more satisfactory learning experience for both students and staff. The use of early diagnostic testing and the option of 'accelerated' and 'standard' programmes also contribute to a learning environment in which the students can follow their own preferred learning styles. Current developments described here include supporting other staff in their own production of screen-capture video, integrating video feedback with on-line self-assessment and developing a web-based learning environment capable of supporting a variety of learning needs.

### Introduction

The applications of mathematics and statistics in science are taught to approximately 300 first year students at the University of the West of England. The students, in forensic, pharmaceutical, and environmental sciences, study a 20 credit module over two semesters, whereas students in the biological sciences study a 10 credit module in the first semester, and develop remaining topics within the context of their biology modules. Most programmes also include a second year module which then develops the statistics of research design leading to preparation for a final year project.

A major challenge with these modules is the very wide diversity of intake. The majority of students have obtained a grade B or C in GCSE Mathematics at least 2 years previously, but the range extends from those with recent A-level mathematics passes, to some (often mature students) with very weak mathematics skills and minimal recent qualifications. However, an additional dimension of diversity is the range of learning styles that suit different students, even those with similar formal qualifications.

The position is further complicated by the fact that the many science students only appreciate the value of 'mathematics and statistics' at the moment that they are faced with data to analyse or a problem to solve. It is difficult to develop motivation for study in anticipation of a need that is yet to be realised.

The developments planned for 2006/2007 were to

- further develop the process of early diagnostic testing to recommend to individual students that they should follow either an 'accelerated' or 'standard' programme of study,
- use combined lecture/tutorial sessions as the principle method of introducing new topics to small groups each week, and abandon the traditional 'resource efficient' one hour lecture delivered to the whole cohort, and

- expand dramatically the use of short video clips as a means of presenting worked answers to the students, and thereby increase the flexibility in the use of different resources.

## **Diagnostic Test**

An initial paper-based diagnostic test of 40 multiple choice questions is used to provide feedback to students on their skills within specific topic areas. This test also identifies the more capable students (approximately one third of the cohort), who are then moved into 'accelerated' groups. At the other end of the scale, this test also identifies deficiencies in very basic maths topics which can then be supported by revision material in both printed and video format.

The overall aim of the diagnostic test is to provide the students with immediate feedback that can help them decide which learning style might be most suited to them, i.e. whether they

- are capable of studying the module with minimal formal class contact;
- could manage to follow an accelerated programme of study to avoid getting bored with the slower pace of the 'standard' delivery;
- should follow the 'standard' programme of study;
- require additional, and more personal, tutorial support; and/or
- should urgently address serious deficiencies in their background knowledge.

The specific test questions have been revised to enable more detailed feedback to the students on their performance, beyond the reporting of a simple overall mark. The students are now also provided with full worked answers on video for all questions.

Once the teaching programme has started, individual students can move, subject to counselling, between 'accelerated' and 'standard' groups depending on their own preferred learning style or current progress.

## **Development of a Learning Environment**

The guiding principle underlying the revision process for these modules has been to develop an environment that will facilitate independent learning and support a variety of learning styles. The ideal elements of a facilitated learning environment are considered to be

- Detailed learning objectives – clear to students & staff (not just a syllabus);
- Clear open perspective of the subject area, which illustrates possible contexts;
- Access to focussed sources of information on demand, and in a variety of forms to suit different learning styles;
- Possibility of studying either independently or within different student groupings;
- Facility for extensive self-assessment, available on demand;
- Provision of immediate feedback on performance.

The overall perspective of the subject is provided by the text book that has been developed within the programme: "Essential Mathematics and Statistics for Science" [1].

The selection of topic areas is defined each week by reference to chapters and sections in the book and the detailed learning objective are specified by over 200 specific ('Q') questions in the book. The definition of the learning objectives are further defined within the classroom situation by using weekly tutorial sheets that reflect the 'Q' questions in the book.

The book itself is now being extensively revised for the second edition in the light of current teaching experience and the new support available via video and other IT resources.

### **Presenting New Material**

The traditional lecture was abandoned because it only suited the learning style of a small proportion of the students, and many students quickly became either bored or 'lost'. The lecture and separate tutorials were replaced by combined lecture/tutorials in which the new material was introduced to smaller groups (of approximately 25/30 students) together with a set of tutorial questions that define the relevant content for the particular session. The answers to the tutorial questions are available online.

The 'accelerated' groups of students attend a one hour session each week, whereas the 'standard' groups attend classes of two hours which allows them to work through example questions as each new part of the topic is introduced. The move to a distributed lecture/tutorial system, with the learning objectives defined separately, has enabled students to develop their own individual learning styles.

All students, after their lecture/tutorial each week, also attend a one hour computer workshops which develop implementation skills in Microsoft Excel [2] and MINITAB [3] that are associated with the new topics.

### **Learning Support including Video**

Access to learning materials, self assessment tests and feedback, is provided principally via the University's VLE. However, much of the interactive material, including flash videos, is held as web-based files on a separate server.

An essential element in enabling the student to develop their own learning styles is the provision of short video clips giving worked answers to:

- 185 questions in the course textbook;
- 80 questions in 8 self-assessment tests;
- 40 questions in the initial diagnostic test;
- 48 questions in two specimen examination papers.

Most of these videos replicate the informal feedback that a student would get if he/she were to call on the tutor to work through a particular problem, and are produced by screen capture on a tablet PC using Camtasia [4] software.

Using these, a very capable student operating almost as a 'distance' learner (i.e. not attending any classes) can pick up quick audio and video explanations of specific questions that he/she may find new and difficult.

At the other end of the learning spectrum, the weak student can pause and re-run the videos at his/her own pace, using them as self-directed tutorials.

Within the more structured context of the module, the students are required to attempt four self-assessment tests. The feedback answers to these (80 questions) are also provided on video. Although the marks do not count directly to the student's overall performance, a student will lose coursework marks if he/she fails to attempt at least three self-assessment tests each semester.

Having opened up the options of other learning styles, the videos perform an essential role in providing the elements of 'communication' between the tutor and students. The students can then use these elements to build their own programmes of study.

The requirement for more personal support is provided in part by a PAL (Peer Assisted Learning) programme whereby second year students work with small groups of first year students. Regular 'drop-in' sessions are also provided by the teaching staff for three hours per week, although the increased use of video has been followed by a dramatic fall in the number of students attending these 'drop-in' sessions.

## Evaluation

Qualitative feedback from the students on the value of the videos has been extremely positive. There were the expected responses based on the flexibility of flash videos, enabling students to pause and 'rewind' the videos to provide study at a pace which suits the viewer. However, it was also pleasing, from a presentational perspective, that they were considered to be 'succinct and precise'.

A particularly interesting observation was that the video answers were 'much easier to follow than a written explanation'. This observation may explain the fact that, although full worked answers to the 'Q' questions had previously been available via on-line pdf files, very few students had accessed any of them.

When asked how these videos could be improved, the students responded that the existing videos were fine in their current form, but suggested that new videos should be made to teach skills in the use of Excel and MINITAB. This is consistent with the known difficulties that some (often mature) students have in the computer workshops with these skills.

Quantitative feedback from 26 students recorded their mean ratings for the perceived values of various learning resources on a scale of 1 to 4:

Video answers to questions from the textbook.	3.58
Video answers to self assessment test questions	3.16
Video answers to specimen examination questions	3.55
Course textbook	3.04
Lectures/tutorials	2.80
Computer workshops	2.24

Table 1: Values (rated 1 – 4) of various learning resources

In response to these results the computer workshops have now been revised and supported by new videos, and additional support material is being developed to help tutors in the lectures/tutorials.

Student feedback within the normal programme quality control has also been very positive, and students now appear to be happy with these 'mathematics' modules whereas previously the feedback was more negative.

## Wider Issues

There are differences of opinion over the need to establish a scientific context in which to teach mathematics to science students, ranging from a 'total immersion' model of mathematics in the scientific topic, to a more traditional separate mathematics course. Certainly it is useful to foster initial motivation by introducing the need for mathematics within suitable applications in science. However, many students then prefer the detailed teaching of the mathematical techniques to be free of context so that they can concentrate on the issues involved, without the distraction of a scientific concept with which they may be unfamiliar. An overemphasis on context can also make it difficult for the students to transfer the technique to other types of problems. Future developments aim to offer flexibility in the introduction of context by structuring optional elements within a web environment.

In addition to addressing the learning concerns of the student, the design and management of these modules has also considered the needs of other interested parties:

- Tutors: As the various tutors are now presenting the new material rather than just responding to tutorial questions, it was necessary to define the requirements rather more closely using the sets of tutorial questions. This appears to be successful, giving the tutors a greater ownership of the material, while allowing them to develop their own teaching styles.
- Institution: The apparent loss of resource efficiency in abandoning the single use of a large lecture theatre has actually been offset by the increased ease of timetabling a number of small classrooms. In addition, there has been no overall increase in the loading on tutors as the use of the video answers has dramatically reduced the steady flow of students repeatedly asking for solutions to the same questions.

## Future Developments

Future developments are following five interrelated themes:

- Involvement of students and other academic staff in the production of video clips and other materials;
- Development of on-line self-assessment that will integrate with video feedback to help students manage their own learning;
- Creation of a web based structure to enable effective use of the new materials, either by individual students with a range of learning styles, or by module leaders wishing to adopt whole sections;
- Revision of the course text book for a 2<sup>nd</sup> edition;
- Use of the technology and pedagogy for topics in chemistry.

Compared with some other web-based materials, screen capture videos are very easy and quick to prepare, requiring training in only a limited range of skills. Other members of staff *and students* at UWE have already produced videos on various science topics, and the feasibility of hosting short training courses in these techniques is now being investigated. Lecturers interested in cooperating in the development of such materials are invited to contact the author with a view to establishing a common interest group.

With the emphasis on supporting a spectrum of learning styles, it is also necessary to improve the availability of suitable methods of self-assessment. Current developments include coupling videos with on-line questions using the simple authoring package, Hot Potatoes [5]. Self-assessment is to be used as both a navigational tool to find the appropriate learning package as well as self-testing following the use of such a package.

The development of the overall learning environment requires a web structure that is capable of presenting the separate learning elements and self-assessments in various learning approaches. For example, some users will prefer to view the materials in terms of their scientific contexts, and a presentation model is being developed in which

- i) case studies could be used to introduce the need for mathematics in science, linked to ...
- ii) detailed teaching with general science-based examples, but with relatively little context, leading to ...
- iii) specific applications in science that the students can use to practice their acquired mathematics skills.

Other users might prefer to be presented with a more traditional 'course' structure that is grouped according to mathematical topics, or, following modern trends, might prefer to access material, on demand, through a 'search' facility.

Screen-capture has now also been used to prepare over 26 videos and self-assessment for a range of topics in organic chemistry as part of the 'Chemistry For Our Future' [6] programme. These videos have experimented with a different, more presentational, style than those used for mathematics, but which is appropriate for their particular context.

## Conclusion

The introduction of screen-capture videos since May 2005 has already made a dramatic change to the learning environment for mathematics and statistics for science students. It has become apparent that the provision of on-line video answers has not only empowered individual student learning but has also enabled staff to produce effective learning materials very quickly.

The enthusiastic response of students has then been the catalyst to develop a wider web-based learning environment, including on-line self-assessment. This environment will provide the structure necessary for students to access the new learning elements within a variety of learning styles.

The focus of course development has moved from a lecture-based delivery of theory and techniques to student-led demand for new structures of support – a form of democratisation of learning.

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## Quantitative skills in the social sciences: Identifying and addressing the challenges

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### Abstract

Social science students are often surprised and dismayed by the requirement to develop statistical skills; many have avoided quantitative work and find the topic to be unfamiliar and frightening. Often these students lack basic numeracy skills and the confidence to develop them, but without quantitative skills their competence as graduates, job seekers and researchers is limited. Many students ask for, and expect they would benefit from, individualized tuition, but contemporary funding models preclude that level of resource for undergraduate teaching, especially in the social sciences where most courses are funded at the lowest level. Lancaster's recent university-wide study has identified two types of students who struggle with quantitative concepts and skills: those who struggle but can face it when encouraged and supported, and those who feel unable to seek help or use the support that is provided. A small set of interactive online statistics-teaching modules, and a system for constructing further modules, is being developed to support the teaching and learning of quantitative skills. A project to develop the system and a few pilot modules is being funded by the ESRC. The principles underlying the system are that students benefit from practice testing with feedback and customized, scaffolded tuition to help students monitor their understanding to guide the development of further understanding as well as the use of clear examples creating a clear purpose for the quantitative concept or skill and demonstrating its usefulness.

### Quantitative skills in the social sciences: Identifying and addressing the challenges

The undergraduate study of social sciences, such as psychology, sociology, geography, politics, criminology, anthropology and social work, all include a research component. The data collection methods often used in the social sciences vary, but are likely to include observation (naturalistic, participant-based and ethnographic), experiments (laboratory and field-based), surveys and questionnaires, diary studies, interviews, focus groups, and case studies. All of these methods can yield data for quantitative and qualitative analysis. Key quantitative concepts covered in undergraduate courses generally include central tendency, variance, statistical and graphical description of a dataset, statistical hypothesis testing, significance levels, basic concepts of probability, Type I and Type II errors, effect size and power. The quantitative analysis tools that are often taught to undergraduates are likely to include:

- sign test, chi-square analysis and log-linear analysis for categorical (or frequency) data;
- *t* test, Analysis of Variance (ANOVA) and its multivariate extension (MANOVA), and correlation-based tools including Pearson's *r*, and linear and multiple regression for normally distributed data;
- Wilcoxon matched-pairs, Mann-Whitney *U*, Spearman's rho, and the Friedman test for non-parametric data.

In each domain the Quality Assurance Agency's subject benchmarks [1] require some degree of engagement with quantitative analysis and reasoning skills. Although the emphasis on quantitative description and analysis methods varies considerably among the social sciences, the requirement exists in all social sciences. Three major issues arise with respect to helping students develop the necessary quantitative skills, which will now be discussed.

### **Students lack confidence and motivation**

Students who choose to study social sciences are often surprised and dismayed by the need for quantitative skills; some see the requirement to develop research expertise, and particularly quantitative skills, as unreasonable and unrelated to their studies [2]. This conflict between expectations and course requirements often leads to motivational problems, which may be aggravated by an *a priori* disinclination to engage with quantitative concepts and skills. Students who have avoided quantitative studies often lack confidence and may feel threatened by the need to build quantitative skills, especially if they have forgotten much that they once knew for a long-forgotten GCSE exam. Students who lack confidence in quantitative reasoning may neglect quantitative methods in favour of qualitative methods. So students' experience with quantitative concepts and skills should build their confidence and provide motivation for developing these skills.

### **Students' a priori numeracy skills vary widely**

Social science students vary widely in their preparedness for quantitative topics. Although many students have avoided quantitative study beyond the minimum required at GCSE level, others have excellent A-level double-mathematics marks. Among the students with the lowest qualifications there are likely to be some with dyscalculia – a learning disorder that often goes unrecognized. This wide variability creates a special challenge for tutors and developers of curriculum materials. If too little scaffolding is provided, weaker students will be left behind, and likely will be demotivated, as described above. If there is too much scaffolding, better prepared students are likely to learn little from it [3], perhaps because it seems too easy; they may also resent the apparent waste of their time. Students recognize that their individual needs differ substantially across a cohort (e.g. [4] and [5]), but it is nevertheless necessary for tutors and curriculum materials to address their varied needs if students are to succeed. The widely variable level of prior achievement presents special challenges for teaching quantitative reasoning and tools.

### **Students' numeracy skills are generally lower than university courses expect**

Many students are substantially unprepared for the quantitative aspects of their course. Some have avoided quantitative work and describe themselves as "rubbish at maths". Bynner and Parsons's [6, p. 103] data suggest that self-report may seriously underestimate the number of students with deficits in their ability to effectively use even basic quantitative concepts and skills, so the skills gap may be worse than students think. Furthermore, modern pre-university education appears to develop relatively lower levels of mathematical fluency than in decades past [7] [8]. Although university tutors may be well versed in the teaching of particular quantitative concepts and skills at the undergraduate level, they may be unprepared to teach basic mathematics, and undergraduate courses typically lack the time and resources to do so. Extra working sessions and surgeries are welcomed by students (e.g. [4] and [5]), and are often provided in the physical sciences and engineering, but social sciences are typically funded at the lowest band, D (except for psychology and geography at band C), by the Higher Education Funding Council for England (HEFCE) [9, see Table 1 for information about funding bands]. Whereas physical sciences, typically funded at band B, can draw upon additional resources to provide surgeries, social sciences cannot. It can be counterproductive to teach undergraduate statistical concepts when tuition in basic mathematical concepts and relationships is needed for many students, but there may not be resources to provide that tuition.

Band	Weighting factor	Example subjects
A	4.0	medicine, dentistry, veterinary science
B	1.7	physics, chemistry, biosciences, engineering
C	1.3	information technology, mathematics, arts, languages
D	1.0	social studies, humanities

Table 1: Summary of the Higher Education Funding Council for England (HEFCE) funding bands, effective 2004-5. The weighting factor is applied to the basic amount that HEFCE pays universities to fund undergraduate students. The complete report is available online [9].

### The minds of the students

To address the needs of social science students with respect to learning quantitative concepts and skills, it is necessary to identify and understand their strengths, weaknesses and perceptions of the relevant teaching and learning environment.

Focus groups with Psychology in Education students at Lancaster University were conducted during the 2006-7 academic year. First- and second-year students were recruited from statistics classes. Sessions were run by two people who did not teach on the course but were familiar with it. Students reported that statistics worried them because they generally understood the gist of their other courses at first encounter, but often did not immediately understand the statistical concepts. They responded to their own confusion with statistics by withdrawing, whereas when they experienced confusion with the other parts of their course they were more likely to seek clarification. Students were especially frustrated by the often greater effort required to understand statistics, because they saw statistics as being simply a matter of “automatic rules,” a view similar to that reported by Ben-Zvi and Garfield [10]. Ben-Zvi and Garfield went on to note that based on this perception of an arithmetic-like approach, students were loathe to engage in interpretation; this observation was appropriate to the students in this programme as well.

Other recent focus groups, surveys and interviews at Lancaster University have asked students to comment on how to make teaching and learning of the quantitative components of their courses more effective [4] [5]. Most students argue for more individual or small group tutorials and labs, many stating emphatically that they cannot learn the concepts and skills from written material. They express frustration at tutors and materials that assume skills and knowledge which they lack – and many argue that they should not be expected to remember what they learned for GCSE mathematics exams. Many students suggest that they need more elapsed time when learning quantitative concepts and skills, explaining that a 10-week term is not long enough to take the material on board. The use of examples was important: Students found the use of real examples useful, but were confused when too many examples were used or when an example was not taken from a clear starting point to a reasonable conclusion. Students were confused by equations and said that they would prefer verbal explanations. They were resistant to the idea that they might learn from a text, but offered some suggestions for making a text more useful. Although they were in favour of assessment within their classes, they argued against formative assessment exercises as being too much work for both tutors and students. In general, weaker students disliked a problem solving approach as being too challenging and preferred working through worked out examples, but recognized that merely following examples might not lead to understanding.

Evidence for the mathematical skills that social science students bring to university was obtained from 77 first year students at Lancaster University who studied Psychology in Education as one of their three subjects; they took an online basic-level mathematics quiz at the beginning of academic year 2006-7. Students were generally competent at many basic arithmetic tasks involving whole numbers, but roughly half of the students made errors when decimal fractions were introduced (e.g., put numbers in order from smallest to largest: .3, .13, .20). They could read graphs reasonably well, but roughly half of the students could not handle very simple algebra questions (e.g., solve for X:  $A + X = B$ ).

## The world of the tutors

For the students, research methods, including quantitative and qualitative analysis methods, are often seen as separate from the content of the discipline [2]. Whereas other parts of the discipline are reasonably likely to be presented in an interrelated way with occasional comments that connect separate modules, the modules addressing other aspects of the discipline rarely make substantial reference to the research methods involved and almost never refer to the types of quantitative analysis employed. Even within modules addressing research methods, it is often the case that quantitative and qualitative methods are taught separately, sometimes as incompatible alternatives. This problem can be exacerbated when the tutor for quantitative skills comes from outside the discipline.

From the tutors' standpoint, there can also be isolation. The social scientists who teach statistics may be somewhat anomalous within their department. In this case tutors may feel the lack of colleagues with whom to discuss the teaching issues that arise and share good practice. Statistics tutors from outside the social science may not have colleagues who share the concerns of the programme or familiarity with the students.

A sense of isolation and frustration can also arise from resourcing issues. Successful teaching of research methods and especially quantitative concepts and skills typically requires different teaching methods and usually requires special rooms (e.g., computer labs) and additional time and teaching staff (for workshops and practical activities). Because most social sciences are funded minimally, there is often strong competition for resources to support teaching. Tutors on modules that are not awarded additional resources sometimes resent modules that are and the tutors who teach them. Alternately, where recognition of the need for additional resources has not been forthcoming, conscientious tutors who provide additional tutorials or surgeries from their own research time may resent their situation.

## A SIMPLE tool to support statistics teaching

Research commissioned by the Economic and Social Research Council (ESRC) (e.g., [11] and [12]) identified a specific need to develop more and better quantitative skills among UK social scientists. The Mills *et al.* report [11] suggests that this training needs to begin at the undergraduate level, and it needs to change the way that these budding social scientists relate to quantitative methods. SIMPLE (Statistics Instruction Modules with Purposeful Learning Emphasis) is one of the undergraduate curriculum development projects funded by a recent ESRC initiative to improve and encourage undergraduate research training.

The project is developing a teaching tool and materials that can be used at an introductory level in any social science department to provide students with effective and engaging tuition that is cost effective and easy to administer and customize. The SIMPLE system includes four pilot modules and the software, examples and instructions to enable tutors to use the pilot modules, modify the modules, or to construct their own. Modules are intended to supplement existing statistics modules, primarily as exercises introduced in class and completed outside of class. SIMPLE is designed to stimulate effective learning for the students, enable tutors to modify or develop modules using familiar file formats and clearly documented procedures, and enable tutors to monitor students' progress and be aware of students requiring special help.

Figure 1 illustrates an example of flow in a SIMPLE module. Each module can be designed to require as much or as little mainstream tuition as the tutor determines is appropriate. Where questions are asked, error handling options include both progressive error handling, as illustrated for the first question in Figure 1, and specific error questions for multiple choice, as illustrated for the second question.

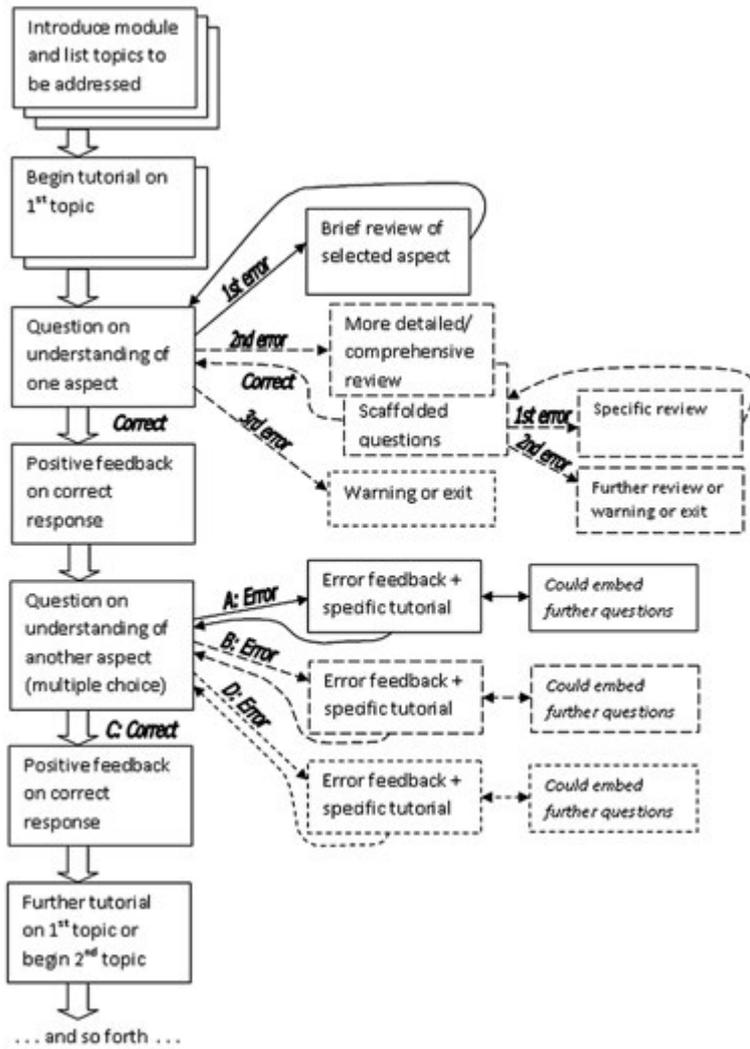


Figure 1: Example flow in a SIMPLE tutorial. Boxes represent one or more slides; block arrows represent main-line flow through the module; line arrows represent optional flow, based on students' responses.

Important characteristics of the modules that stimulate effective learning include:

- Combining instruction with practice, which informs subsequent instruction. The practice questions provide diagnostic and formative assessment. Where students show a lack of understanding, the modules provide simpler instruction, as an individual tutor would. Where students succeed, the modules move along. Tutors can design parts of modules to allow students to select a path through the module based on their own assessment of their prior knowledge. It is possible to offer students with substantial prior knowledge an initial set of questions so that students who perform well can skip the module or select only parts they need. At the other end of the spectrum, it is possible to offer students who lack prerequisite knowledge access to appropriate tutorial and reviews of that knowledge. In this way, each student receives individualized instruction.
- By asking questions at early stages and monitoring students' profiles, the modules help students to build confidence and learning skills. Carefully thought out, well designed modules avoid unwarranted assumptions about students' prior knowledge. Modules may ask students if they want a review of a basic topic before moving on, allowing students to guide their own learning and encouraging them to monitor their understanding. Each screen can include links specified by the module designer to supporting materials (e.g., slideshows, webpages).
- The modules can be introduced in class, where students might obtain help getting started, but they can be completed from any computer linked to the internet. This practice allows students to work at their own

pace. This flexible approach, combined with online resources, can also encourage students to work in small study groups even when it is not convenient to be in the same place. Students can share comments, questions and support via email or synchronous discussion tools external to SIMPLE. In this way, SIMPLE encourages the development of cooperative work and effective learning skills.

Tutors interface with the system in developing or modifying modules and in monitoring students' progress:

- Modifying modules, especially the examples, is easy. The original materials are mostly PowerPoint slides, and tutors merely need to change the context of the example to adapt a module developed for one discipline to fit squarely within another. In many cases, though, this adaptation may not be necessary as the original examples are intended to engage the students by addressing student issues.
- The development of modules is not easy from a conceptual standpoint: The designer must imagine a plethora of one-on-one tutorials, anticipating students' errors and misunderstanding and designing appropriate interventions for these imaginary students. But having done that, the development of materials is relatively simple in that it requires no special, new or unfamiliar skills. In its simplest form, a module can be built from PowerPoint slides and an Excel file to direct the flow of the module and provide links to supporting material.
- The system maintains records of students' progress for tutors to review and alerts tutors whenever a student has exhausted the tutorials available in the system without appearing to understand the topic.

Pilot modules have been developed and trialed at Lancaster University during 2007-8. The system and modules will be available to all interested parties during the summer of 2008.

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# How should we prepare Mathematics undergraduates for the workplace?

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## Abstract

Developing workplace skills, as part of the undergraduate student experience, is of increasing importance to Universities throughout the UK. The introduction of student fees were justified, to a great extent, by the higher earning potential of graduates relative to those who do not participate in Higher Education. Although vocational degrees have always focused on employment outcomes, many other degree programmes are now liable to be judged, in part, on their ability to deliver graduate level employment opportunities.

Mathematics degrees may be strongly placed to benefit from an increased focus on employment outcomes. Many employers value mathematics graduates for being highly numerate and possessing problem solving and analytical skills. However, employers also emphasise the importance of other skills such as team working, project management and presentation skills, outcomes that have not always been closely identified with undergraduate mathematics programmes. Mathematics graduates may well find themselves in competition for jobs with numerate graduates from disciplines such as engineering and computer science who have had opportunities to develop a wider set of employability skills.

This paper arose from a working group brief at The Undergraduate Mathematics Teaching Conference (2006) to investigate examples of current practice in Mathematical Sciences undergraduate programmes for delivering graduate employment opportunities.

## Introduction

Integrating transferable skills effectively into undergraduate programmes is an issue that has become a priority for Higher Education, stimulated by a number of key reports on the Higher Education sector. The Dearing review of Higher Education [1] and the DfES White Paper on Higher Education [2], both emphasise that higher education has an important contribution to make to the economic and social well being of the nation by preparing future graduates with the necessary skills to make an effective contribution to the workplace. The Roberts Report [3] identifies

*"mismatches between the skills of graduates and postgraduates and the skills required by many employers"*

and goes on to state that

*"many [graduates] have difficulty in applying their technical knowledge in a practical environment and are seen to lack transferable skills".*

Mathematical Sciences graduates are generally viewed as being in a strong position in the graduate employment market. The QAA Benchmark statement for Mathematical Sciences [4] claims that

*“students who graduate from programmes in MSOR have an extremely wide choice of careers available to them”*  
and

*“All employers know that MSOR graduates possess knowledge and skills that will enable them to make a contribution that is beyond the capabilities of those without a background in MSOR.”*

Current evidence [5] does support the first statement that graduates of mathematical sciences find employment in a wide range of economic sectors. The second statement makes an assumption about the skills that MSOR graduates possess. The reference to an MSOR background could include graduates from engineering, science, economics or computing, who would all be sufficiently numerate for most employers. While we would confidently predict that a mathematics graduate would possess a more in-depth knowledge of mathematics or statistics than an engineering graduate, what would we conclude about problem solving skills, team working, presentation skills or report writing? Accepting the role of Higher Education in producing a skilled graduate workforce, the Royal Society [6] expressed concern about levels of recruitment to UK first degrees in technology and mathematics and the impact this will have on the UK Government’s widely stated aim of maintaining the UK’s position as a leading knowledge economy.

The premium that graduates enjoy with respect to their earning potential during their working life over non-graduates was a widely publicised argument in favour of the introduction of student fees. It is therefore reasonable to assume that student satisfaction in the years following graduation will be dependent upon this predicted graduate outcome.

The Prospects website [5] consistently places the starting salaries of mathematics as being higher than the average for all graduates. In 2003, the average starting salary for mathematics and informatics graduates based on a survey of advertisements in the Graduate Prospects’ weekly vacancy bulletin *Prospect Today* is given as £19,202 (average for all subjects is £18,175). However, the returns from the Graduate Destination Survey shows that six months after graduation, the average salary of mathematics graduates was substantially lower at £16,911 (average for all subjects was £16,393). The returns for 2005 show that the average starting salary for mathematics graduates was £19,342 compared to the average for all subjects of £17,715. So it appears that the earning power of mathematics graduates is currently increasing possibly due to the increasing importance of the financial sector in providing jobs for mathematics graduates.

It is difficult to say what impact student fees will have on student degree choice. Students will probably continue to give high priorities in their choice of degree subject to factors such as interest in the subject, course structure and location and reputation of the HE institution. However, Hibberd [7] has shown that mathematical sciences students give a high priority to personal development and transferable skills. However, the subject specific profiles for mathematical sciences [8] suggest that the development of employability skills has not been made explicit within mathematical sciences programmes. Hibberd and Grove [9] have argued for a more explicit integration of personal development and transferable skills into mathematics curricula.

In this investigation, we report on different strategies for incorporating activities within mathematical sciences undergraduate programmes, to promote awareness of the employment market and develop skills that are valued by employers. We then look at examples of good practice for integrating personal and transferable skills into mathematical sciences curricula, consider resources that can be used to develop awareness of the employment market for mathematical science graduates, and conclude with some recommendations that will hopefully stimulate further discussion and interest in this important issue.

## Curriculum Design

The subject benchmarking for Mathematics describes the skills profile of a typical MSOR graduate [4]. Although possessing strong cognitive skills, the profile suggests our degree programmes do not adequately provide a chance to develop personal and transferable skills and organisational awareness. However there are examples of good practice in developing a wide range of graduate skills in the curriculum in Mathematics across the HE sector in the UK, which help to promote independent study, research, written and oral communication and problem-solving skills.

- Project Work – a survey of all mathematics departments in the UK [10] revealed that the majority of departments offer opportunities for undergraduates to undertake some project work during their programme.
- Group Projects – working with their peers on a mathematical project helps students develop an awareness of team roles and dynamics and provides experience of peer assessment.
- Work-based Experience – provides students with opportunities to develop organisational awareness, adapt to new situations and understand the relevance of mathematics to real-life situations.
- Enquiry Based Learning – covers a whole range of activities where the learning is student led. It encourages research skills, independent learning and the ability to solve new and unfamiliar problems.
- Personal Development Plans – allow students to reflect on their learning and skills development and devise action plans for effective study. They help students deal with time management and provide advice for developing graduate skills.
- University Careers Services – provide a great deal of support for graduate skills development. This may involve undergraduate course units aimed at improving personal and professional skills and enhancing awareness of graduate careers.
- Peer Mentoring and Peer Assisted Learning – pair first year students with more experienced undergraduates to provide academic and pastoral support. As well as the obvious benefits to first year students the mentors develop skills in group facilitation, communication and management.

## Examples of Good Practice

For a general contact for further information on the examples discussed in this section email:

Kevin.Golden@uwe.ac.uk

### University of Manchester

#### (i) Peer Assisted Study Sessions (PASS)

PASS is a student led scheme run by 2<sup>nd</sup> and 3<sup>rd</sup> year student mentors. The mentors are trained in group facilitation and student learning. In a weekly class, the mentors help first year students to work on their mathematics problems. They also offer advice and support in a friendly, informal setting. Mentors develop a range of skills in leadership, team working and communication.

#### (ii) Careers Skills Course

This course helps students prepare for job applications and is run by the University Careers Service. As well as interview technique and CV writing, there are presentations from some of the top graduate employers.

Assessment is via group presentations and a portfolio. The module offers students a chance to develop their communication skills and business awareness.

More information: [louise.walker@manchester.ac.uk](mailto:louise.walker@manchester.ac.uk)

## University of Chester

### Experiential Learning

Second year students at Chester undertake either a work-based learning module or a module entitled 'Experiential Learning' in which they complete a series of tasks. In recent years these have included:

- working as a team to prepare and/or send out a mailshot for the department;
- running programmes on Matlab to support the research of the department;
- mathematical modelling;
- working as a team to produce a handbook for prospective students.

Meetings of module tutors and students are held about once a week but the students are responsible for managing their time to complete the work allocated and are expected to seek guidance when needed.

More information: [p.lumb@chester.ac.uk](mailto:p.lumb@chester.ac.uk)

## University of Nottingham

### Vocational Mathematics

The third year group-project module 'Vocational Mathematics' includes skills workshops and project activities taken as a compulsory module for MMath (Maths with Engineering) or optionally for other Mathematics students. A major aspect is to develop skills and to provide experience of the organisational, technical and self/peer assessment requirements of project teamwork. The module is synoptic in bringing together the subject-specific knowledge and mathematical skills attained in the first two years of the course and developing mathematically relevant 'graduate' skills:

- analysing open-ended problems to a tight deadline;
- selecting and applying mathematics;
- working collectively and delegation in groups (teamwork);
- communicating quantitative ideas orally and written reports;
- integrated use of IT.

An electronic Personal Evidence Database (PED) is used to enable students to further synthesise the skills developed within the module, to build up individual reflection and data on their activities and attainments. The PED also provides an awareness of the employment aspects together with examples of typical interview questions.

Two assessed group projects give students the experience to work in different teams, and through oral presentation, submission of reports, peer assessment activities and detailed feedback including video playback of group presentations, provides an environment to learn from experience and from each other.

Feedback from student surveys conducted between the start and end of the module confirm a significant increased confidence in their graduate skills.

More information: [Stephen.Hibberd@nottingham.ac.uk](mailto:Stephen.Hibberd@nottingham.ac.uk)

## University of the West of England

### Project work

Students are introduced to project work in their first year through two modules, Mathematical Problem Solving and Statistical Problem Solving. Students work on a number of group-based case studies, producing reports, posters and presentations. Group work activities continue into the second year. All students take an individual project in their final year. The current scheme with project and case study work being made core in each year of the programme has been in operation for two years. In addition to the problem solving skills that mathematics students are expected to develop throughout a mathematics degree, students are able to develop a number of transferable skills, such as being proficient with different styles of presentation, report writing and project management. It is too early to assess the impact of this activity on the quality of final year projects or employment outcomes, but first year student cohesion and retention has noticeably improved over the last two years.

Students have three choices of individual project in their final year. These are a mathematics project, a quantitative methods project and a mathematics education project. Students on the mathematics education project are participating in the University Ambassador Scheme, where they spend time working in a local school. Their project, which typically involves research into some aspect of pedagogic theory, culminating in the design of learning materials which are then used in the delivery of a mathematics lesson, is carried out in the school under the guidance of a teacher. Assessment of the project is through the keeping of a logbook, a written project report and a presentation. The module provides an important opportunity for students to find out if a teaching career is suitable for them.

More information: [Kevin.Golden@uwe.ac.uk](mailto:Kevin.Golden@uwe.ac.uk)

## Sheffield Hallam University

### Online Progress Files

Online Progress Files are used as part of students' Personal Development Planning (PDP). Instead of being a stand alone element, the Progress Files are embedded within the mathematics curriculum. This helps students appreciate the relevance of recording and reflecting on their progress and helps them to become more effective and independent learners. The Progress Files consist of three components: a Logbook where the student regularly records progress in particular modules; a Problem Diary where the student records problems as they arise and progress towards their resolution; a Personal Website where the student records work done for each module and an up to date CV.

More information: [N.Challis@shu.ac.uk](mailto:N.Challis@shu.ac.uk) or [J.A.Waldock@shu.ac.uk](mailto:J.A.Waldock@shu.ac.uk)

## Placement Opportunities

A number of universities offer sandwich degrees whereby students can choose to break their academic study with a year in industry or business, usually taken in their third year at the university. Obvious benefits to a student in taking a placement include the boost in self-confidence, discipline and responsibility, the economic benefit of paid employment for a year, and the experience of working for a potential future employer. A work experience survey carried out by the University of Manchester and UMIST in 2004 [11] reported that

- 69% of placement students were offered graduate level jobs;
- 80% of employers recruited placement students with the primary aim of attracting them back to permanent jobs;
- 40% of annual graduate intake from these employers consisted of former placement students.

Academic benefits included the consolidation of theoretical knowledge within a practical setting and the formation of ideas for final year projects. A number of studies [12-14] have reported significant improvements in academic performance that can be attributed to having taken a placement year. Although the above studies did not involve students taking degrees in the mathematical sciences, there is no reason why mathematics students would not benefit similarly.

## **Awareness of Employment Opportunities**

There are many online resources to help students discover the range of careers taken up by mathematics graduates.

- Maths Careers [15], developed by the IMA the LMS and the RSS, provides comprehensive advice on how mathematics is used in everyday life and details common career paths of maths graduates together with useful links to other careers websites.
- Prospects [16], provides career information, postgraduate study and further training. The graduate destinations survey provides data on employment sectors for mathematics.
- Plus [17] is an online mathematics magazine that includes an extensive list of case studies of mathematics careers.
- Maths Jobs [18], lists vacancies available to mathematical science graduates and provides insight into specific skills requested by graduate employers.

## **Summary**

Graduate outcomes in terms of employability, is likely to become an increasingly important measure of the success of a university education with benefits to the individual graduate and to society at large. A considerable amount of effort is being made to measure graduate outcomes and to report on shifts in patterns of graduate behaviour on an annual basis. The Royal Society [6] has welcomed the intention of HESA to augment the six month destination survey with a longitudinal study taken three years after graduation.

Mathematics graduates have a reputation for finding employment in a wide number of employment sectors. This view is backed up by the evidence collected from the Graduate Destination Survey [5]. However, by and large, the employment market for mathematics graduates is dominated by the financial sector (20.6% in 2005) and teaching (7.1% in 2005). The other common outcome, is studying for a higher degree (10.1% in 2005).

There is evidence that the mathematics community in Higher Education is responding to the issue of incorporating transferable skills into mathematics undergraduate curricula. A number of examples that integrate transferable skills into traditional mathematical activities can be found in different universities, which have the potential to enhance the educational experience of a student.

Activities that raise awareness of the employment opportunities open to mathematics graduates and the development of the necessary skills and techniques to make effective job applications, may not sit too easily alongside academic activities that are competing for time and space within the student timetable. However, it is reasonable to assume that students themselves will demand support for their personal development and progress towards finding suitable graduate outcomes on completion of their degree. Returns from the HESA

Graduate Destination Survey and the HEFCE National Student Satisfaction Survey, both provide mechanisms for the comparison of performance of different disciplines across Higher Education and opportunities for students to make their views clear to HE providers.

Perhaps the most direct example of addressing an employability agenda is through workplace learning, be it a placement in industry or business, or a school-based placement. This not only provides a student with the opportunity to gain experience of working in a particular sector, it also allows them to develop maturity, skills and attitudes that are of benefit when they return to University. Rawlings *et al.* [14] demonstrate, in the context of information systems students, that the effect of a placement year could be shown to have significant positive impact on the final year performance of students from a wide range of academic abilities as measured by their second year performance, not just the most able.

## Recommendations

We conclude with the following recommendations for future activity:

- Personal development, transferable skills and employment awareness should be built in mathematics curricula;
- Project work with a variety of means of communicating results should be encouraged in all years;
- Students should be made aware early on in their studies of the range of employment opportunities that are open to them;
- The MSOR network through mini-project funding, good practice guides or workshops could promote good practice in the integration of employability skills into mathematics curricula.

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# Introducing Logic

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## Abstract

In this paper, we address three issues concerning the teaching of logic. First, we consider some key themes arising in the introduction of logic to students of mathematical sciences. Second, we discuss, in particular, how to deal with the culture shock often experienced by students during the transition from pre-university work to the first year of an undergraduate programme. Third, we offer some useful observations, some practical suggestions, and some informative examples for teaching and learning.

## Introduction

This paper considers some matters arising in the teaching of logic to students of mathematical sciences, with a particular mention of issues related to the construction of proofs. Our main concerns are problems arising in the transition from school/college to university, and issues involved in the teaching of first year undergraduate Mathematics (level four in the national qualifications framework). However, many of the topics on which we touch are relevant also to the general mathematical community, including secondary educators, and, indeed, to the perception of Mathematics held by the general public as a whole.

The paper provides a brief introduction to what we believe are the key issues and offers what we hope will be useful observations and suggestions for teaching. For an overview of the large existing body of literature concerning pedagogical research on logic and proof, we refer the reader to the MSOR booklet by Nardi and Iannone [1], to the book by Nardi [2], and also to the website of the *International Newsletter on the Teaching and Learning of Mathematical Proof* [3].

This paper does not discuss:

1. the teaching of logic as a topic in its own right (this would usually constitute a level seven module in a mathematical sciences programme);
2. the application of notions related to the logical foundations of Mathematics to proving specific theorems (for example, in algebra or in analysis).

## What do we mean by 'logic'?

We use a rather loose definition, involving an appropriate blend of the following overlapping themes:

- the nature, practice and communication of Mathematics;
- the basics of predicate logic, introduced as a tool for *doing* Mathematics;

- some 'foundational' mathematical *notions*.

We now expand on each of these three themes.

### **The nature, practice and communication of Mathematics**

If we consider the general practice of Mathematics, then most mathematicians would agree that their fundamental job is to prove - and perhaps then to apply - theorems, and that it is the notion of rigorous proof which separates Mathematics from all other spheres of human endeavour. However, this view is not the general public perception of Mathematics, and neither is it necessarily the view of Mathematics held by students who are about to embark on a study of mathematical sciences at university. Indeed, unless a student has taken A-level Further Mathematics, for example, it may be that they believe that the heart of Mathematics is concerned with manipulation or calculation, rather than with formulating and proving results. Even for well-prepared students, the only experience of mathematical rigour might be proof using mathematical induction. Thus, one of the major rôles of the university teacher is to ease the students' transition from school/college to university, both by showing why mathematical proof is necessary, and by equipping the students with the fundamental logic and language skills necessary for constructing and communicating proofs in an efficient way at the appropriate level. Here, 'appropriate level' is difficult to define, but usually this means that the proof should be written out in such a way which balances correctness and pedantry, and which convinces both the student and those with whom they are communicating (e.g., themselves at a later date, their fellow students, or their tutor).

Before embarking on a quest to instil in undergraduates those ideas of logic and language required for the construction of proofs, it is first necessary to persuade these students that proof is actually needed in the first place. In particular, students have to be persuaded that proofs are just as essential for 'obvious' statements as for 'non-obvious' ones. This process of building awareness of proof and logic is part of a general introduction to mathematical culture, and perhaps it could be aided by suggesting - or requiring! - that first year undergraduates read some accounts dealing with the *modus operandi* of Mathematics (of which there are several, including books by Gowers [4] and by Singh [5] and an article by Krantz [6] which can be accessed from his website). This awareness building presents us with some difficulty: on the one hand, many of today's students have been brought up in a culture where few things are allowed to demand their attention for very long, but, on the other hand, mathematicians are trying to demand perseverance, discipline and stamina. We need to inform students that work in Mathematics is usually done via some kind of 'investigative' approach - possibly taking quite a while! - which, in the end, might lead to a conjecture. Only when equipped with a proof, will the conjecture become a theorem.

The persuasion process might start off with suggestions such as 'once you know for sure that there is no largest prime number, you may rest easy in your bed at night, and those terrible nightmares should become a thing of the past', or 'once you know that this computer programme definitely does what it's supposed to do, selling your software will make you a millionaire'. On a more serious note, it should be pointed out that proof is at the heart of predictability in Mathematics. For example, once you have proved to your satisfaction a formula for the sum of the first  $n$  natural numbers, then you can apply the formula with confidence to finding the sum of the first billion natural numbers; no longer must you add up these numbers by hand. The issue here, of course, is agreeing exactly what amounts to 'satisfaction' in this concept of proof. Students will arrive at university being perfectly satisfied with a level of convincing which professional mathematicians might consider inadequate. It probably does not help to mention that what has constituted a proof has varied over time, but students do become confused when people in different situations appear to demand different standards of rigour. We will never entirely change this, but at least the individual lecturer can take care in applying a consistent standard of rigour, even though that may occasionally require discipline. Some honesty here may help: we cannot prove everything - so what do we accept? Do we need to prove that  $6 \times 7 > 5 \times 8$  before we use that fact?

After an induction into the general nature of mathematical culture, students need to be exposed to a collection of results, which can form the basis of discussion, and which provide an arena in which to apply specific

logical methodologies. They also, of course, need to be successful in their introductory steps if they are to be encouraged to proceed much further. There are two main sources of such results. Firstly, theorems can arise as part of the natural progression of first year courses, and these can be presented directly to the students. Secondly, conjectures can arise via investigations performed by the students themselves. There are several types of theorems or conjectures which might be considered, the choice depending on the experience of the students or on the particular course in which the material is being developed. For example, results which provide an explicit formula are especially useful in this context, but other possibilities include: proving that a given object is an element of a particular set; existence or uniqueness results; structure results (e.g., results of the form 'the set of all such objects has the structure of a vector space'). In any case, perhaps the best examples involve infinite sets, for example the standard number systems or sets of geometric objects, because it is only when infinite sets are considered does Mathematics truly become differentiated from other areas of human activity. However, it can be pointed out to students that, even when the domain of discourse is finite, you might not want to check every case by hand, or even by using a computer!

Another issue is that of student engagement. Broadly, students will not learn much of lasting value unless they end up doing things themselves, and appreciating that they want to do them for more than just the course assessment. We need to tread carefully, and ensure that students are soon successful in proving things. In this respect, topics related to number theory are useful in that students have little difficulty with the concepts; they think they know what integers are. This allows interesting results on factorisation, primes, sums of squares and the like to be used as vehicles for teaching proof. Students whose main interest is in the application of Mathematics may, however, be less appreciative of this sort of result (although some motivation might be provided by pointing out the importance of number theory in areas such as cryptography and internet security).

### **The basics of predicate logic, introduced as a tool for doing Mathematics**

At the start of a university programme, predicate logic should be presented in a *much* more relaxed way than would be done in a course on formal logic, and the ideas should be introduced via interesting mathematical examples (see below). Good sources at the correct level for this material include the books by Eccles [7] and by Velleman [8]. It is worth emphasising, in particular, the following basic areas.

- The rôle of axioms.
- The rules of inference - all of which originate from *modus ponens* or universal generalisation - and their use in the construction of the various types of proof. Special mention should be made of direct proof and of proof by contradiction. The Deduction 'Theorem' should be mentioned explicitly, and simple examples of its use should be provided.
- The crucial importance of quantifiers in Mathematics, e.g., for constructing definitions (and - equally importantly - their negations), and for stating conjectures/theorems. Note that, once the significance of quantifiers is properly understood, there is much less chance of confusion related to issues such as 'proof by example', 'general versus specific', the use of counterexamples, etc.

Partly in connection with the second and third topics in this list, the method of exhaustion should not be forgotten, and this could be linked to the use of technology with reference to such historically important examples as the proof of the Four Colour Theorem.

### **Some 'foundational' mathematical notions**

During the first year of a university programme, at least some of the following general ideas should be introduced (or perhaps reintroduced), not only as vehicles for logic, but also because they are very important in their own right.

- The natural numbers: the Axiom of Infinity; the Principle of Induction and the Well-Ordering Principle; the Recursion Theorem (the proof of which is not entirely trivial!);
- Divisibility: the division algorithm; the fundamental theorem of arithmetic;
- Basic combinatorics.

### The rôle of examples

The examples used to illustrate mathematical methodology and the tools of predicate logic should enjoy at least some of the following qualities.

- They should be related to the students' own experience: pre-university; other parts of their university programme; investigative work (perhaps involving the use of computer software).
- They should be interesting, surprising, or important (perhaps because of subsequent courses which the student will take).
- They should feature some aspects which highlight common student stumbling points.

Some good examples which can be used to illustrate the methodology of Mathematics and the tools of predicate logic include the following.

#### i) There do not exist positive integers $p, q$ such that $p^2 = 2q^2$

This is a historically important result, and it is crucial for the development of Mathematics. It shows the need to extend the system of rational numbers; 'the quest for closure'. There are links with ideas concerning approximation, and with the representation of numbers by computers. The result requires notions of divisibility, the use of preliminary lemmas, and it can be proved in several ways.

#### ii) There does not exist a largest prime number

This is another historically important result. The result shows that a natural subset of the natural numbers is, in fact, infinite. It is a starting point for number theory.

#### iii) The sum of an arithmetic series

Fill in the 'gaps' in the proof produced by Gauss (aged seven) of the formula for the sum of an arithmetic series. Students will probably have seen this during their pre-university days. This exercise can be approached at several levels of sophistication. Indeed, at a later stage, it could be used as a vehicle to introduce more advanced concepts such as: induced  $n$ -ary operations; generalized associativity; invariance under the symmetric group; ...

#### iv) The Collatz Dynamical System

This is the discrete time dynamical system on the natural numbers  $\mathbf{N}$  induced by the map  $\varphi: \mathbf{N} \rightarrow \mathbf{N}$  defined as follows:

$$(\forall u \in \mathbf{N}) \left( \varphi(u) = \begin{cases} \frac{3u+1}{2}, & u \text{ odd} \\ \frac{u}{2}, & u \text{ even.} \end{cases} \right)$$

(See, for example, [9].) In addition to providing an introduction to the theory of dynamical systems, an investigation of this map provides a rich source of individual or group investigations using computer software

(e.g., *Excel* spreadsheets, *Maple* programmes, ...). In term of the themes of this paper, we see in action the Recursion Theorem, the Well-Ordering Principle and the Division Algorithm. An investigation of the Collatz system provides ample opportunities for the formulation of interesting conjectures (with, of course, the associated precise use of quantifiers). Students may construct - and carefully communicate - proofs for some of these conjectures (but perhaps not for all of them ...).

## Conclusions

In this paper we have considered some of the issues related to the teaching of logic to students of the mathematical sciences. In addition to identifying the underlying concerns, we have provided some approaches to address these, and we have made some practical suggestions which the reader may wish to use in their own teaching.

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# Implementing graph theory into Mathletics

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## Abstract

The development of the Mathletics database of online objective questions has so far been concerned with algebra, calculus and statistics. Many question types, additional to the usual multi-choice and numerical input, have been created to help students to obtain maximum benefit from the question, especially via the generally extensive feedback. Each question realisation is generated by coding a question style that incorporates random parameters in all elements (wording, equations and diagrams). This technology and pedagogy has also been exploited for other disciplines, especially in physics and economics. However, unlike the existing topics, graph theory does not dwell so much on numerical calculation as on visual interpretation and understanding. As a result, developing online objective question styles for graph theory raises new and rather different issues.

Central to the coding is Scalable Vector Graphics (SVG) that allows random parameters to be carried through to the graph(s) displayed. Randomised parameters within the coding thereby allow different graphs to be realised, based on arbitrary incidence or network matrices (different connectivity and weights) of arbitrary size (different numbers of vertices). Using a set of external functions to handle the generation of SVG strings automatically therefore frees the question setter to concentrate on the underlying pedagogy of the question.

This paper describes how graph theory has been incorporated into Mathletics by exploiting SVG. Different question styles and types, including a new question type, namely Random Word Input, are reviewed to see how they can generate valuable questions for use in graph theory. Examples from basic graph theory and more advanced topics, such as degree sequencing or chromatic numbers, are presented. Issues of input checking, display clarity and mal-rules and their effect on feedback screen design and marking are discussed.

## Introduction

Graph theory is a subject at the university level that serves many purposes in various fields, especially within mathematics, economics and business management [1]. At Brunel University many of the basic elements, such as cycles, subgraphs, and trees, are taught fairly quickly before moving on to more advanced topics. Given that students may not have taken Decision Mathematics 1 (D1) at A-level [1], there seems to be a need for a set of questions, including randomised elements so that each question *style* generates thousands of realisations of the question at runtime, on which students can practice to secure the basics. This paper describes its development within Mathletics [2].

In graph theory, there are two ways of representing graphs: **visually** using graphs, and **numerically** using adjacency matrices. To present graphs visually, scalable vector graphics (SVG) is used, but to accommodate the desired randomness, adjacency matrices must be used. Nonetheless, issues still arise with determining how to design a randomised graph with a random number of vertices, such as:

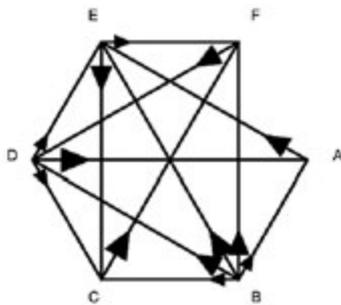
- How can random graphs of  $n$  vertices be generated so that all vertices will definitely appear on the screen?
- How can loops in a graph be drawn?
- How should undirected or directed graphs be encoded?
- Where does the randomness of these graphs come into play?

**Technical issues**

In order to create randomised graphs of any size and with any valid adjacency matrix (which is chosen by the question setter to have the required structure – e.g. bipartite -but otherwise random) within any web-based assessment engine or, indeed, ordinary web pages, we have created a set of functions that return appropriate SVG strings that are used by SVG Viewer (a browser plug-in). The question setter is then free to focus on the pedagogy of the question and, especially, the quality of the feedback, simply calling such functions as black boxes. Given that SVG syntax is not widely known we present the *annotated* code of some parts of the digraph function in the Appendix.

A full listing of all functions is available [3]. As an example, Figure 1 shows a digraph with arrows given by  $ratio\_along\_line = 0.2$ .

Consider the following graph:



input the edge set in alphabetical order, each vertex being separated by a comma only, e.g. AB,AC, \_AZ,BA,EB,EC, \_BZ...  
 Edge set = { }

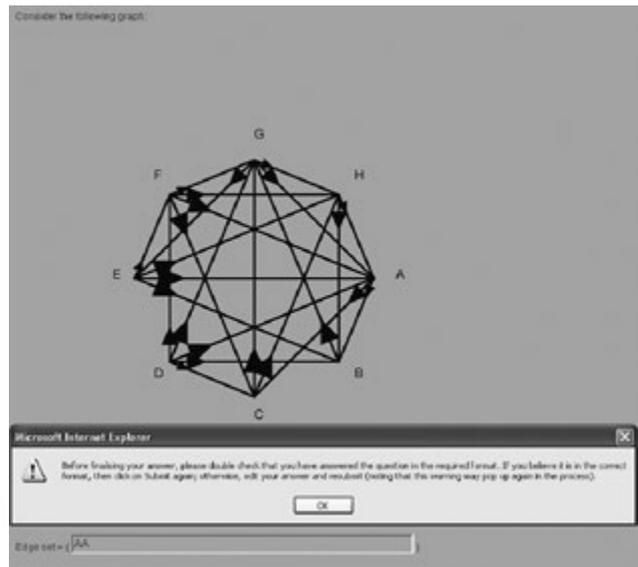


Figure 1: Two realisations of the same Responsive Word Input (RWI) question asking for the edge set of the digraph (in alphabetical order). In the second realisation, the student has answered inappropriately, triggering the alert and allowing re-input.

**Pedagogic issues**

The question shown in Figure 1 is a Responsive Word Input (RWI) question. In this question, the correct answer is a set of items in a string, with each item in the list separated only by a comma, as specified in the question. Because so much has to be typed in completely accurately, the student’s input format is checked before marking to avoid penalising and hence frustrating the student. Such checking can take account of the string length, and order of input in some cases and allows the students to re-input their answers. If the check fails, the student is

allowed repeat attempt(s). Figure 1 asks for the edge set; similar checks can be implemented in questions that require e.g. a path to be specified.

This question is also responsive, in that solutions arising from logical (or at least structured) but incorrect methods (mal-rules) are encoded within the question. Thus if any of these mal-rules are used by the student, then the student is told not only that they are wrong, but why they may have gone wrong. See Figure 4 for examples of this within different question types.

Another feature of the coding highlights loops formed with two edges connecting the same pair of vertices. Having such a feature can be useful for students new to graph theory, see Figure 2.

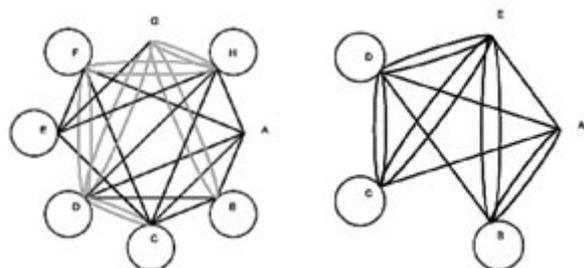


Figure 2: Two realisations with loops formed by two edges between the same pair of vertices and with: a) coloured loops (shown here in grey, these vertices appear red on screen), and b) loops not coloured.

In Mathematics, all previous multiple choice questions were designed in a cyclic pattern of correct answer and distracters 1- 4. However, with much of graph theory being visual, keeping this cycle may lead to students spotting it and thus getting the marks without doing the question. To resolve this, the ordering of the display of answers has been altered using a shuffling method. As shown in Figure 3, not only will the answers be shuffled, but distracter 4 can also appear with the correct answer (rather than replacing it as previously, when 'None of these' is correct).

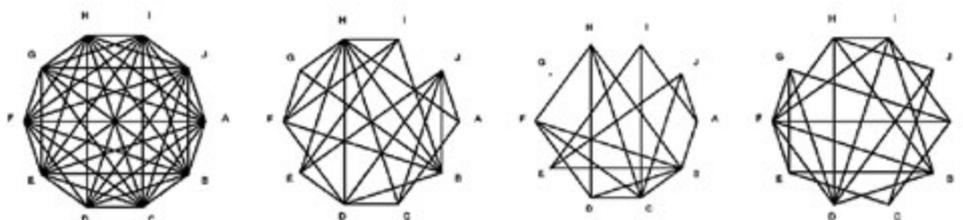


Figure 3: An (annotated) multi-choice question on planarity, giving shuffled options as either graphs (shown here) or adjacency matrices.

The distracters in multi-choice and responsive numerical input questions are based on mal-rules. Normally, such mal-rules are based on students' exam scripts. However, the present topics are not covered in much detail at Brunel University and thus are not normally assessed. Instead, the mathematical theory behind these topics is analysed. For example, in Figure 3, the question is asking to find a graph that is planar; the distracters must therefore be non-planar. In order to create a non-planar graph, we use Kuratowski's Theorem, which states that a simple graph is planar if and only if it does not contain a subgraph homeomorphic to the complete graphs,  $K_5$  or  $K_{3,3}$  [4]. Using this, distracters are formed by including either or both subgraphs into a graph. Another mal-rule is set to be a complete graph of  $n$  vertices, where  $n \geq 6$ , with one vertex missing, see Figure 3, distracter 3.

## Marking issues

Depending on the purpose of the test (diagnostic, formative, summative) marking may or may not be an issue. In fact, the marking schema is independent of the content of the questions and feedback; generally outcomes

based on student's responses should be attributed marks on the basis of the test's purpose, perhaps offline. In Mathematics, usage so far has stressed formative assessment so most answers tend to be worth one mark if correct, zero otherwise. Next year these graph theory questions will be used summatively with second-year students, so marking needs to be considered much more carefully. For instance, the question in Figure 3 will take a lot of effort if the number of vertices is large. Therefore, this question is scored using a scoring judgement system [5], where a correct answer is awarded 4 marks, an incorrect answer is awarded 0 marks, but responding with "I don't know!" will earn 1 mark. This should help to remove some of the guessing by students, but the marks awarded for not knowing the answer should probably be only 10% - 20% of the full mark so that students do not make a habit of selecting this answer.

Partial and negative marks can be awarded to some options, depending on the difficulty of the question and how mal-rules differ from the correct answers. In addition to giving the full solution, the feedback coding also identifies the students' probable errors and marks awarded appropriately, depending on the merit of their answers. For Figure 4a, the submitted answer is judged to be less seriously wrong than that for Figure 4b, where the student is penalised; again this should deter guessing.

A particular graph has the chromatic polynomial,

$$P_G(k) = k^6 - 6k^5 + 14k^4 - 16k^3 + 9k^2 - 2k$$

Determine the chromatic number for this graph.

a) Your answer, 2, isn't correct. It should have been 3.

The polynomial,

$$P_G(k) = k^6 - 6k^5 + 14k^4 - 16k^3 + 9k^2 - 2k$$

has order, 6. Therefore, there are, at most, 6 roots to the equation. For chromatic polynomials, though,  $k \geq 0$  for all  $k$  and thus, there are  $k$  real roots, some of which may be the same as others.

To determine the chromatic number for this polynomial, first calculate  $P_G(0)$ , i.e.  $k = 0$ . If  $P_G(0) = 0$ , then increase  $k$  by 1 and recalculate. Keep doing this until you find one positive integer such that

$$P_G(k) \neq 0$$

This value of  $k$  will be the chromatic number for the graph represented by the given chromatic polynomial.

In this question, the 6 roots for the given polynomial are  $k=0, 1, 1, 1, 1, 2$ . Therefore, the chromatic number for this polynomial is 3.

---

**Related material**

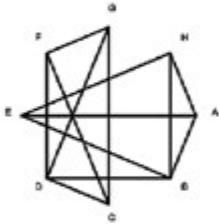
1 out of 2

You are on the right track, but you may have calculated the highest integer value of  $k$  for which the polynomial is equal to 0, but you need to find the smallest, positive, integer value of  $k$  for which the function is not equal to 0.

**SOLUTION**

This is a popular problem within the category of Hamiltonian cycles. Recall that a **Hamiltonian cycle** is a cycle that goes to each vertex once and then returns to the initial vertex. In this problem, we want to find the adjacency matrix corresponding to a graph that does NOT have a Hamiltonian cycle as it.

The only one of these four adjacency matrices to not have a Hamiltonian cycle is representative of the graph shown below, which, if you look closely enough, is, in fact, an **isomorphism** of the complete bipartite graph,  $K_{6,2}$ . It can be shown that all complete bipartite graphs,  $K_{x,y}$ , where  $x$  is not equal to  $y$ , do not contain Hamiltonian cycles.




---

**Related material**

-1 out of 2

Another adjacency matrix was selected. This adjacency matrix resembles a complete graph of  $(n-1)$  vertices with one vertex added with two edges connected between it and any of the other  $(n-1)$  vertices. This, however, should obviously contain a Hamiltonian cycle. Therefore, you will receive a negative score for this question.

Figure 4: Examples of feedback from a Responsive Numeric Input (RNI) question.

Up until now, all graphs have been formed by placing all vertices around a circle. However, this is not useful for some special graphs, such as the wheel and ladder graphs. In the isomorphisms topic, it is important for students to see what these graphs look like in order to have some idea as to how to find an isomorphism for it. As such, two new functions, namely SVG\_wheelgraph and SVG\_laddergraph, have been added to accommodate this, see Figure 5. Features of this question's stem include the learning material and the random choice of presented graph (type and number of vertices).

Andrea is a university student at University of Michigan. Recently, she has been learning the following:

**n-prisms** ( $P_n$ ) are graphs with  $2n$  vertices and  $3n$  edges that create two similar shapes of  $n$  vertices joined by the corresponding (matching) vertices between them.

**n-antiprisms** ( $AP_n$ ) are graphs with  $2n$  vertices and  $4n$  edges that create two similar shapes of  $n$  vertices, where each vertex of the bigger shape is connected to two adjacent vertices in the smaller shape.

**Wheel graphs** ( $W_n$ ) are graphs where  $n$  vertices are joined to its neighbouring vertices to form a cycle and one additional vertex is placed in the center. This vertex is connected to all of the  $n$  vertices, thus creating the spokes of a wheel.

**Complete bipartite graphs** ( $K_{x,y}$ ) are graphs with  $n$  vertices and composed in two sets of  $x$  and  $y$  vertices. Each vertex in set  $x$  connects with all vertices in set  $y$  and vice versa. However, there are no edges connecting vertices in one set to other vertices in the same set.

**Ladder graphs** ( $L_n$ ) are graphs with  $2n$  vertices and composed in two sets of  $n$  vertices each. Vertices in each set are connected only to one adjacent vertex within the same set. However, each vertex in one set is connected to a corresponding vertex in the other set, thus creating a rectangular structure composed of squares, almost like using matchsticks to draw squares connecting to each other.

Andrea has been asked to find a graph that is isomorphic to 3-prism. However, she has found four incidence matrices that are potential candidates and is unsure which one to select.

Which of these incidence matrices should Andrea select?

Click on the input of your selection at the bottom left of the graph. You may also choose either *None of These* or *I Don't Know* if you prefer.

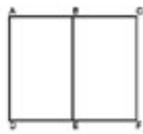
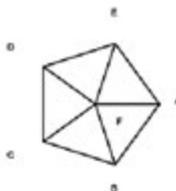


Figure 5: Two examples of isomorphism question stems: a) a ladder graph and given additional information about the different graph types, and b) a wheel graph without any additional information given.

Clare is a university student at Lakehead University. Recently, she was asked to find a graph that is isomorphic to the wheel graph shown below. However, she has found four potential candidates and is unsure which one to select.

Which of these adjacency matrices or graphs should Clare select?

Click on the input of your selection at the bottom left of the graph or adjacency matrix. You may also choose either *None of These* or *I Don't Know* if you prefer.



### Conclusions

By exploiting random parameters with SVG, it is seen that setting objective questions for graph theory within a computer-aided assessment system is feasible. It is anticipated that the variety of question types and full and responsive feedback will prove helpful to student learning. A new marking scheme is designed to encourage student engagement with the material and to make guessing less attractive. Partial and negative marking may be used to give a more perceptive reflection of students' understanding of the material. Much of this pedagogy, and all of the programming, should prove useful to further development within and beyond the Mathletics system.

### References

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## Appendix

The annotated code below is used to display any SVG graph of  $n$  vertices by means of keeping all vertices equidistant from each other in a circular form. The circular placement of all vertices is performed on the complex plane, where  $z = x \pm yi$ .

```
function SVG_digraph(A,ratio_along_line,filled,double_path_colour,double_path_skinnyess,svg_start){
// this function produces SVG code for the digraph with adjacency matrix A, with given arrow position, arrows filling, double paths
being represented by skinny ellipses and (optional-specified) canvas and viewBox size

var fs = getFontSize()/16;

size1 = fs*800; // and similar for other sizes that are resized according to the student's choice of font size to accommodate disabled
students

if(svg_start == null){svg_start = '<iSvg:svg height='"+size1+"' width='"+size2+"' viewBox="0 0 '+size3+' '+size4+'"><iSvg:g
id="canvas">';} // this sets the canvas and viewBox sizes, see [6]

for(k = 0; k <= n-1; k++){

  x_coord[k] = r*Math.cos(2*k*Math.PI/n)+offset;

  y_coord[k] = r*Math.sin(2*k*Math.PI/n)+offset;

// this sets the coordinates of each vertex equally spaced around a circle, similarly for their label coordinates and for vertex loops...

colour = getFgColor(); // this reads the students chosen font colour from a cookie

for(i = 1; i <= n; i++) {

  SVG_graph += SVG_ellipsebl(x_coord[i-1],y_coord[i-1],0.1,0.1)+SVG_scale_text(x_coord_label[i-1],y_coord_label[i-
1],alphabet(i-1,1),colour,1); // this draws the vertex dots and labels them

  for(j = 1; j <= n; j++){ if(A[i][j] == 1){

if(ratio_along_line != 0){SVG_graph += SVG_arrow(x_coord[i-1],y_coord[i-1],x_coord[j-1],y_coord[j-1],ratio_along_
line,filled);}else{SVG_graph += SVG_line(x_coord[i-1],y_coord[i-1],x_coord[j-1],y_coord[j-1]);}}

// this constructs the single edges either as arrows or lines by calling other functions; those "atomic" functions handle the resizing
and recolouring according to the student's choices

  if(A[i][j] == 2){ // this detects double edges in the adjacency matrix: various definitions follow ...

SVG_graph += SVG_ellipse_rotate(cxx,cyy,rx,ry,theta,double_path_colour);}

// this detects and concatenates skinny ellipses to the SVG string to draw double edges

  if(A[i][i] == 1){SVG_graph += SVG_ellipse(x_coord_loop[i-1],y_coord_loop[i-1],rrloop,rrloop);}

// this detects and concatenates small circular loops to the SVG string

return svg_start + SVG_graph + svg_end;} // this returns a long string comprising canvas and content
```

# Computer games and mobile technologies: Motivating students towards mathematics learning

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## Abstract

It is widely accepted that many university students experience difficulties in learning mathematics or mathematics related subjects. This paper aims to provide an overview of the current “state of the art” in the use of new technologies in mathematics learning with particular reference to computer games and mobile technologies. These in particular are believed to have the potential of motivating students towards their mathematics learning. Possible uses of games and mobile technologies are explored by citing examples of recent research into and evaluation of the educational use of these new technologies, as well as the learning theories which underpin the integration of new technologies into teaching and learning. Different types of mobile technologies are introduced, their educational potentials are considered and related pedagogical issues are discussed.

The paper also identifies possible future trends of new technologies, such as educational multi-user virtual learning environments, and possible different ways of supporting mathematics teaching and learning through new technologies with reference to the “Web 2.0” paradigm shift.

Finally, conclusions are drawn about the way in which computer games and mobile technologies can empower, mediate and motivate students to learn mathematics, both in formal and informal settings, and how teaching strategies could be changed.

## Introduction

Many students have difficulty learning mathematical subjects and are not motivated to study mathematics. According to the Robert’s report, the numbers taking A-level mathematics in England fell by 9% between 1991 and 1999 [1]. Some university students are also struggling with mathematics in various subject areas partially due to inadequate school preparation. These echo the findings of an international survey on relevance of science education [2] which indicates a decline in the recruitment of students to science and technologies studies. A crisis in mathematics education has been recognised. The emergence of new technologies, including computer games and mobile technologies, introduces potential new ways of teaching and learning mathematics and new ways of motivating students to learn mathematics.

Both mobile technologies and games technologies are increasingly seen as fertile ground for the development of resources to support learning [3]. Mobile technologies offer the opportunity to embed learning in a naturalistic environment, enabling learning to happen at any time and anywhere, while computer games are interesting because they are potentially capable of offering a variety of ways to learn with varying degree of difficulty, leading to a ‘flow’ of interaction and engagement [4]. They can also facilitate learning environments with peer-collaboration and the social construction of knowledge [5]. Winnicott describes the relationships between

play and creativity: "in playing and only in playing, the child or adult is free to be creative" [6, page 53]. Most importantly, games promote engagement by combining play and creativity in an interactive learning process; such engagement captures students' attention and is an essential element in teaching and learning.

Computer games can also enable immersion in a mixed-reality environment that augments both physical and social space and some have the potential to provide a more motivating learning experience [7].

Roschelle [8] identifies the potential of wireless mobile learning devices to achieve large-scale impact on learning because of portability, low cost and communication features.

The integration of mobile technologies and computer games into teaching and learning mathematics is a novel methodology. Research into the pedagogy and the impacts caused by introducing these new technologies in teaching and learning of mathematics in Higher Education (HE) is in its infancy. In order for the potential of integrating new technologies in teaching mathematics to be fully realised, it is important to understand the current state of the art and the underlying pedagogical principles.

### **The deployment of computer games in mathematics education**

In order to address the gap in the computer games market of few mathematical computer games suitable for university students currently being available, mathematical games currently being used in secondary school education are first examined so that their potential benefits for HE can then be extrapolated.

In 1992, the Electronic Games for Education in Math and Science (E-GEMS) collaborative project, based at the University of British Columbia in Canada, began to explore the potential of specially designed computer games for children aged between 10 and 14, to increase learning and appreciation of mathematics and science [9]. The project finished in the late 1990s, and two computer games called Super Tangrams and Phoenix Quest [10] were developed and evaluated.

The design of Phoenix Quest attempted to address the perception that most computer games were biased towards boys because they contain a lot of action and violence (see [11] for a detailed analysis of this issue). It emphasises three primary game elements: a story, interactive communication between the player and the story characters, and mathematical puzzles covering a wide range of concepts that are embedded in the story context. Visual aesthetics not only offers learners the experience of beauty but also motivating elements. In their evaluation they discovered that Phoenix Quest strongly appealed to girls but boys liked it as well [11].

Available mathematics learning resources increasingly include computer games. The BBC bite-size revision material for GCSE courses provides a series of games on different subjects, which can be downloaded onto some mobile phones [12]. When playing a game, the tutor will give instant feedback with a very friendly tone. If you win the game, you are praised; otherwise, you are encouraged to try another one.

In a recent Summer School at a school in West Yorkshire, a group of 26 11-12 year-old pupils were observed. They appeared to be completely absorbed in a collection of web-based mathematical computer games and other computer-based mathematics learning resources called *MyMaths* [13], designed for children aged 11-16. *MyMaths* contains clusters of games at different mathematics levels, covering algebra, geometry and statistics. In the Summer School, on the day of the observation the pupils played the games for 5 hours and solved problems on the given worksheets. The tutor announced the correct answers at the end of each session, let the pupils mark their own work then recorded the results. Generally, at least 80% of the pupils' work was satisfactory and they were shown to be capable of playing mathematical games. The teacher has been using this website for his mathematics teaching for three years and has found it very useful, complementing textbooks. In the computer games, mathematical concepts are visualised. The tutor uses games in classroom for up to 20% of the total teaching time.

At West Nottinghamshire College in the UK, a modified computer game called Neverwinter Nights [14], originally produced by BioWare, is used to improve students' mathematics skills, and is especially aimed at disaffected students. In the two years since the games were introduced, about 700 learners have played them, and achievement of key skills had trebled to 94%, according to the BBC news on 12 January, 2007 [15].

From 2003, an international collaborative project led by the London Knowledge Lab, as part of the Kaleidoscope project and funded by the European Network of Excellence [16] which comprises over 1000 researchers at more than 90 units spanning 25 countries across Europe, plus Canada, and has a mission of shaping the scientific evolution of technology enhanced learning, has been investigating the educational potential of computer games and the difficulties of designing and deploying games for mathematical learning at the schools level. Its outputs included a literature review [17] [18] and a prototype of a computer system known as MoPiX [19] that allows the user to program games and animations with equations. In the review, the value of games was found to be in increasing the motivation of students, in promoting the empowerment and autonomy of students over their learning, and in supporting constructivist models of learning.

So far, some computer games used for secondary education have been examined. More examples of mathematical games are given in Table 1.

Title	Year	Maths Content	Level of Maths	Genre	Age range	Sources
Dimensionian	2006	Algebra	GCSE	Action adventure	11 -- 14	<a href="http://www.dimensionm.com">www.dimensionm.com</a>
Lost Mind of Dr. Brain	1995	Spatial mathematical thinking	GCSE	Puzzle	12-up	<a href="http://www.mobygames.com/game/lost-mind-of-dr-brain">www.mobygames.com/game/lost-mind-of-dr-brain</a>
Math Missions	2003	Arithmetic	pre-GCSE	Adventure	8 -- 12	<a href="http://www.edutainingkids.com/reviews/mathmissionsgr35amazing.html">www.edutainingkids.com/reviews/mathmissionsgr35amazing.html</a>
MaxTrax	2002	Arithmetic and Algebra	GCSE	Sport simulation	16-up	<a href="http://www.aqua-pacific.com/education.htm">www.aqua-pacific.com/education.htm</a>
MyMaths	2002	Algebra, Geometry and Probability	GCSE	Puzzle	11 -- 16	<a href="http://www.mymaths.co.uk">www.mymaths.co.uk</a>
Neverwinter Nights (modified)	2002	Basic maths skills	GCSE	Adventure	11 -- 14	<a href="http://nwn.bioware.com">nwn.bioware.com</a> , modified by the West Nottinghamshire College, UK
Phoenix Quest	1994	Fractions, number series, logic and 2D geometry	GCSE	Puzzle	9 -- 14	<a href="http://www.cs.ubc.ca/labs/egems/phoenixquest.html">www.cs.ubc.ca/labs/egems/phoenixquest.html</a>
Super Tangram	1994	transformation	GCSE	Puzzle	10 -- 14	<a href="http://www.cs.ubc.ca/labs/egems/supertangrams.html">www.cs.ubc.ca/labs/egems/supertangrams.html</a>

Table 1: Examples of mathematical computer games

Can computer games shape the future of supporting mathematics learning at university-level? Recent literature and the current practice in schools strongly indicate the educational value of using computer games within education [4]. In mathematics education, university students are required to adopt a more self-regulated approach with self-reflection to foster higher order thinking than during their secondary education. They are expected to make links between previous knowledge and new knowledge and to make links between concepts and procedures. To cater for university students' interests and their learning goals, when designing or selecting a computer game for teaching and learning mathematics, two factors should be considered:

1. the need to build students' confidence and enable students to rehearse existing skills with encouraging feedback, and
2. the need to make the context of games relevant to young adults.

### Mobile technologies for learning mathematics

Mobile technologies can be useful for supporting mathematics teaching and learning. Sharples *et al.* [20] offer a theoretical framework on mobile learning (see Figure 1). It describes how learning occurs within a social-cultural system, in which many learners interact to create a collective activity framed by cultural constraints and historical practices; learning is mediated by tools that both constrain and support the learners in their goals of

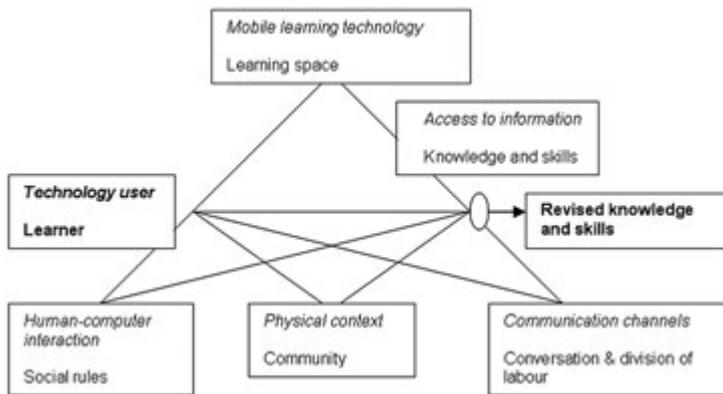


Figure 1: A theoretical framework for analysing mobile learning (Adapted from Sharples, M. et al., 2005)

transforming their knowledge and skills. In the framework, the mediation can be analysed from a technological perspective (in *italics*) of human-computer interaction, physical context and digital communication, as well as from a human perspective of social conventions, community, conversation and division of labour. These two perspectives interact to promote a co-evolution of learning and technology.

Changes of lifestyle in the UK mean that learners are increasingly on the move to different locations, taking ideas and learning

resources gained in one location and applying or developing them in another. Considerable learning therefore occurs outside the classroom. Furthermore, the ubiquitous use of personal and shared technologies is a driving force for mobile learning. In the UK, by 2003 over 75% of the general population and 90% of young adults owned a mobile phone [21]. Mobile technologies are therefore an ideal platform to support learning, including the learning of mathematics.

The variety of types of mobile device currently available offers different ways to learn mathematics. Some observers believe that a major convergence of mobile technologies is in progress [22]. They anticipate a future ubiquitous single type of mobile device with multiple functions, such as broadband internet, a mobile phone, a multimedia computer and a digital camera, which will benefit learners.

The main types of mobile device which are currently available are:

- **Mobile games consoles**, such as the PlayStation Portable, have limited display capabilities, but it is sometimes possible to use Flash to create virtual, on-screen keyboards for use in mathematical assessments.
- **MP4 players**, such as the video iPod, which enable the download and playing of video files. They can also be used to play games, to view teaching material and for assessment.
- **Personal Digital Assistants (PDAs) and Palmtop Computers**, such as the HTC Advantage and the HP iPAQ. These are hand-held computers, usually running a cut-down version of Windows, which provide a variety of data processing applications, such as word-processing and spreadsheets.
- **Smart phones**, especially 3G mobile phones with broadband internet such as the Nokia N70. This is Java-enabled and capable of running simple mathematical games and displaying mathematical graphs.
- **Ultra Mobile PCs (UMPCs)** – a platform specification for small (screen size no bigger than 7 inches) tablet computers that are capable of running a full version of Windows XP. Examples are the Samsung Q1 and the Sony Vaio UX.

Table 2 provides a mapping of a variety of potential uses of these different types of mobile device in mathematics education. UMPCs are the most versatile type of mobile device currently available. They can handle all forms of use except messaging but including operating virtual learning environments and collaborative tablet working. Smart phones and PDAs / palmtops have similar potential uses and are quite versatile. MP4 players are more limited but are still capable of displaying mathematics and therefore may be useful for computer aided learning in mathematics. Mobile games consoles have the most limited use in addition to their intended use.

	Mobile games consoles	MP4 players	PDA's / Palmtops	Smart phones	Ultra Mobile PCs
Capturing audiovideo			✓	✓	✓
Computer assisted assessment	✓	✓	✓	✓	✓
Computer assisted learning		✓	✓	✓	✓
Displaying mathematics		✓	✓	✓	✓
Messaging			✓	✓	
Playing games	✓	✓	✓	✓	✓
Playing videos		✓	✓	✓	✓
Tablet communication					✓
Virtual learning environments					✓

Table 2: Mobile devices and their potential.

### Pedagogy, theories and debates

When using new technologies for teaching and learning, pedagogical issues need to be reconsidered. The heterogeneous mathematical abilities in the student population and the unique learning culture of the “Net generation” [23], give rise to the introduction of new teaching methods to overcome the limitations and constraints of university degree courses taught in a traditional way, which tends to be a passive learning experience for many students. New technologies can offer a wider range of activities for teaching and learning and enable active and collaborative learning; they can also provide excellent tools for teaching. For example, some educational software can effectively visualise abstract mathematical concepts like gradient.

If new technologies are the solutions, what learning theories will guide the new educational practice? The well-known learning theories relating to game-based learning and mobile learning are as follows:

- **Social-constructivism [24]**

This emphasises intrinsic learning through social interactions and accepts the plurality of meanings. From this viewpoint, learning is a process of constructing knowledge and enhancing the ability and strategies to learn, rather than a process of receiving, digesting and reproducing rote knowledge delivered through prescribed curricula.

- **Social-cultural theory [25]**

This focuses on the causal relationship between social interaction and individual’s cognitive development, derived from Vygotsky’s Zone of Proximal Development. This zone is defined as the distance between the level of actual development and the more advanced level of potential development through the interaction between more and less capable participants, so that the less capable participants can achieve what are beyond their competence when acting alone.

- **The experiential learning model**

This can be described as a learning circle (see Figure 2). This theory was formulated by David Kolb, situated with the constructivist learning approach founded by Jean Piaget. The experiential approach stresses the connection between concrete experience, reflection, concepts and application. This learning circle fits well with the playing of computer games, as computer games offer such concrete experiences that can be reflected, conceptualised and applied continuously [26].

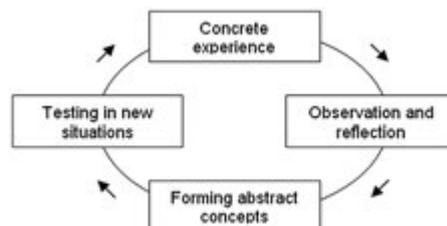


Figure 2: The experiential learning model. These four elements are the essence of a spiral of learning that can begin with any one of them, but typically begins with a concrete experience.

- **Situated Learning Theory**

This theory considers that learning as it normally occurs is a function of the activity, the context and the culture in which it occurs. Social interaction is a critical component within this theory. For this perspective, knowledge needs to be presented in an authentic context, such as settings and applications that would normally involve that knowledge [27].

Both computer game-based learning and mobile-learning share the same notion: knowing and doing are linked and learning should occur in rich, authentic, and purposeful contexts. In particular, social interactions, collaboration and knowledge construction happen naturally in a dynamic mobile learning environment. Wireless technologies like Wi-fi and Bluetooth can allow students to learn through a network of handheld devices. Furthermore, the Internet makes online gaming and mobile networked learning a reality, offering a greater platform for mathematics teaching and learning.

An example based on these theories is the educational use of multi-user virtual environments, such as The River City Project, enabling learners to achieve physical, social and symbolic distribution of cognition [5].

Mobile devices have mobility, flexibility and availability. It is possible for learners to share their learning process and outcomes without being in the same physical place. However, mobile devices have some constraints like limited memory and CPU size, small screens and low bandwidth; therefore, the formats of learning material need to be tailored.

There are debates about the role and practices of game-based learning and mobile technologies. It is argued that they should be exploited in educational contexts, rather simply believing that they are motivating and fun and using them unquestionably. As Egenfeldt-Nielsen [26] indicates, games provide superficial information – not enough to satisfy young people's educational needs, but enough for them to get a grasp on it.

The future trends of mobile learning could take advantage of the Web 2.0 paradigm shift in the internet and its associated technologies [28], including social networking tools such as *Facebook* and *MySpace*, the social construction of knowledge using wikis such as *Wikipedia* and internet picture and video sharing services such as *Flickr* and *YouTube*. Within this paradigm, the web is regarded as a platform and data as the driving force. This could lead to having more control and flexibility in their learning.

## **Conclusions and Future work**

Computer games are an integral part of the social life of children. Lepper and Malone [29] found that computer games have certain features in common: they provide a feeling of control in children and make them curious and provide both intrinsic and extrinsic fantasies, and they challenge children. It is no doubt that these qualities will also appeal to adolescents. For these reasons, it is possible to use computer games for instructional activities. As Klawe [10] points out, computer games have a number of advantages because of their interactive and multimedia capabilities, and their ability to keep students deeply immersed and engaged for a long time. She also indicates that perhaps due to such an immersive effect, students often fail to be conscious of concepts, structures and algorithms they encounter and use in computer games; moreover, they fail to transfer what they have learnt to other contexts.

Computer games can be highly effective in encouraging students' learning and enjoyment of mathematics. However, many factors could affect the effectiveness of game-based learning, such as interface style, the level of integration with other learning activities, and gender differences. When introducing computer games for mathematics teaching at universities, many factors, such as the relevance to the curriculum, the setting and pattern of use, and teacher and student expectations, should be taken into account. Mobile gaming takes advantage of computer games and mobile technologies; it can be a new way of supporting learning mathematics and other subjects on the move.

Mobile technologies have penetrated the world with great depth and speed. The challenges for universities now are multidimensional: First, to develop didactic environments, tools and learning materials for mobile devices; second, for teachers to integrate them into their teaching and learning strategies; and third, to develop new criteria for assessments.

All in all, we believe that the use of computer games and mobile technologies can motivate students towards mathematics learning.

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## **Biomathtutor: what is it and can it help?**

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### **Abstract**

*Biomathtutor* represents a prototype multimedia e-learning resource, which aims to support mathematics learning in the biosciences. It incorporates video mathematics tutorials, similar in format to those pioneered successfully in *mathtutor*, but the latter's more traditional mathematics teaching and learning model has been adapted to deliver a learning resource more directly relevant to the needs of bioscience undergraduates. A contextual, scenario-based problem-solving learning model has been adopted, in which a case study scenario, which covers aspects of haematology and microbiology, is presented via a stimulating high quality professionally produced narrated film. Linked to the content of the film are thirty-three interactive questions for students to attempt on screen. Twenty-four additional practice questions, which cover the same range of basic mathematical concepts, presented in similar biological contexts, are also available for completion. In addition, students can access five relatively short face-to-face video mathematics tutorials in which a tutor explains some of the mathematical concepts students encounter in the interactive questions.

During 2006/07, a project funded by the Higher Education Academy assessed the quality of the prototype learning materials developed and investigated their potential for integration into bioscience curricula. The methodology adopted involved the analysis of both quantitative and qualitative data from undergraduates and their tutors, using questionnaires and follow-up focus groups (students) and interviews (tutors). Overall, the reactions of students and their tutors toward *biomathtutor* were very positive, with both groups commenting favourably on aspects of its design and its potential to support mathematics learning.

### **Introduction**

*Biomathtutor* represents a prototype multimedia e-learning resource, aimed specifically at supporting mathematics learning in the biosciences, and is the culmination of collaborative work involving a group of academics and the Educational Broadcasting Services Trust (EBST), funded by the National Teaching Fellowship Scheme. The design of such a learning resource represents a timely initiative in response to growing evidence of a mathematics skills deficit among increasing numbers of bioscience undergraduates [1-4], a problem exacerbated by the increasing diversity of science entrants' pre-university qualifications.

In designing *biomathtutor*, the decision was taken to adopt the technologies used and expertise developed in the production of *mathtutor*. But rather than adopt *mathtutor's* traditional mathematics teaching and learning model, *biomathtutor* would adopt a contextual, scenario-based problem-solving learning model. This model would be supported by providing students with a stimulating film-based bioscience case study scenario and by incorporating computer-assisted formative assessment (via scenario-linked questions and extension practice questions) in an attempt to motivate students to *want* (rather than merely *need*) to learn the mathematics they would encounter within their bioscience curricula [5-9]. In addition, the teaching and learning of some basic

mathematical concepts would be delivered and supported by a series of face-to-face video tutorials, similar in format to those incorporated in *mathtutor*, but much shorter and bioscience-focussed in content.

There is a long tradition of using contextualised problem-solving to support undergraduate teaching and learning across the many disciplines that comprise the biosciences [10], but its adoption in mathematics learning support represents a departure from many of the models currently used to deliver mathematics teaching. The more traditional approaches to teaching mathematics are believed by many to be inappropriate for the majority of bioscience undergraduates because they fail to take account of the students' lack of confidence in, and their often profound anxieties concerning, anything mathematical. The aim of *biomathtutor* is to capture students' interest and curiosity by presenting them with a filmed bioscience-based scenario and to use the latter to gently guide students to the mathematics they need to understand and grow confident and competent in using routinely.

*Biomathtutor* comprises the following four main components, with free navigation throughout:

### Case study film:

A case study scenario, which covers practical aspects of haematology and microbiology, is presented via a stimulating high quality professionally produced narrated film (24 min in length). The film introduces Rebecca, an A-level student, who visits her GP with the symptoms of anaemia and a mouth infection. Her GP takes a mouth swab for analysis and sends Rebecca to the hospital for a blood test. The film illustrates a hospital laboratory where a full analysis of Rebecca's blood sample is carried out, and a microbiology laboratory where the cause of Rebecca's mouth infection is identified. The film concludes back in the GP's surgery where Rebecca receives her test results and is prescribed appropriate medication.

### Case study questions:

Linked to the content of the case study film are thirty-three interactive questions for students to attempt on screen. Nine questions are related specifically to haematological aspects of the case study, while the remaining twenty-four relate to the microbiological sections. Students receive immediate feedback on their answers and the questions are linked directly to associated maths tutorials providing additional guidance.

### Extension questions:

Twenty-four additional extension (practice) questions, which cover the same range of basic mathematical concepts presented in similar biological contexts, are also available for completion (Fig. 1).

Figure 1: A screen grab illustrating an example of an extension question, with feedback, in *biomathtutor*

## Maths tutorials:

Students can access five relatively short (5-10 min) face-to-face video mathematics tutorials in which a tutor explains some of the mathematical concepts students encounter in the interactive questions; topics include powers, SI units, nomenclature, cell volume and percentage.

During 2006/07, a project funded by the Higher Education Academy assessed the impact of blending *biomathtutor* with more traditional teaching methods, with a view to supporting mathematics learning in the biosciences. The project sought to answer key questions concerning the quality of the prototype resource developed and the potential for integration into bioscience curricula.

## Methodology

The methodology adopted involved the collection and analysis of both quantitative and qualitative data from twenty-seven undergraduate bioscience students and eight of their tutors. The students were predominantly Stage 1 undergraduates, although a small number of foundation year and pre-university students also participated.

Two paper-based questionnaires were designed, one for completion by students and the other for completion by their tutors. Both questionnaires were similar in terms of content with minor re-wording of questions to specifically address either student or tutor. The questionnaires consisted of seven sections, each dealing with a separate component of *biomathtutor* or addressing specific concerns that students or their tutors might have: (1) overall design of *biomathtutor*, (2) *biomathtutor* as a learning tool, (3) the case study film and interactive questions, (4) the extension questions, (5) the tutorials, (6) general likes and dislikes (open questions), and (7) attitudes toward mathematics (for students) or integrating *biomathtutor* into curricula (for tutors). In addition, interviews were held with individual tutors and focus group sessions were held with students. The interview and focus group schedules, while following the same themes contained in the questionnaires, were semi-structured, allowing participants to discuss any other issues they had not previously raised.

## Results

### Design of *biomathtutor*

Overall, tutors and their students felt that *biomathtutor* was very well designed. Participants agreed that they could navigate easily through the learning resources and that the different sections were easily accessible.

*"Clear what they're [students] doing and how they're doing it. Stand alone – which is useful."* (tutor)

Some felt that a restructuring of *biomathtutor* was necessary with regard to the order of the sections through which users were expected to work. Tutors and students alike suggested that all the sections should be more explicitly linked together so that each of the four sections (case study film, associated questions, extension questions and tutorials) had links to the others. It was also pointed out that once a student left the case study film to answer some questions and to view the tutorials, he/she could not easily return to the precise section of film they had been viewing.

Tutors experienced some technical difficulties when attempting to transfer the contents of the DVD to their institutions' intranets or VLEs and had to enlist the help of technical ICT staff. Since most problems had been resolved before students came to use *biomathtutor*, few students indicated that they had experienced technical difficulties. However, the issue of ensuring that the learning materials were readily accessible to students with disabilities, e.g. visual or hearing impairments, was raised and will need addressing in any further development of the *biomathtutor* resource.

### Biomathtutor as a learning tool

Students felt that *biomathtutor* represented a good learning resource and most students felt it helped them to acquire new knowledge, to increase their competence and, for some, to increase their confidence in mathematics. Many students agreed that they would use *biomathtutor* again.

Tutors generally agreed that *biomathtutor* would enhance their students' knowledge in both the mathematics and bioscience topics it covered and therefore this resource could be integrated into their curricula. Tutors felt that this resource had the most to offer to Stage 1 students and that it was not particularly relevant for students below that level (i.e. pre-university or foundation year). Tutors did agree that *biomathtutor* could be appropriate for use by students in Stages 2 and 3, but primarily as a revision tool.

*"It sets the 'boring' topics into context so I am hoping it will engage the better students early in the module as well as instruct the weaker ones."* (tutor)

### Case study film and all questions

Overall, students liked this section of *biomathtutor*, agreeing that the film was enjoyable, informative and enhanced their knowledge. They also liked the associated interactive questions, believing that the opportunity for additional practice provided a valuable learning experience and the feedback on answers was also helpful.

*"Questions were related to my science course - like calculation on haemocytometers."* (student)

The case study film and questions were also very well received by tutors. Overall, tutors agreed with students that the case study film and questions provided a valuable learning experience that would reinforce students' understanding of maths.

### Video maths tutorials

Of the five tutorials, the one explaining powers was the tutorial most viewed by students; however, this may have been because it was the first in the list of five tutorials. Figure 2 summarises the students' views of this tutorial.

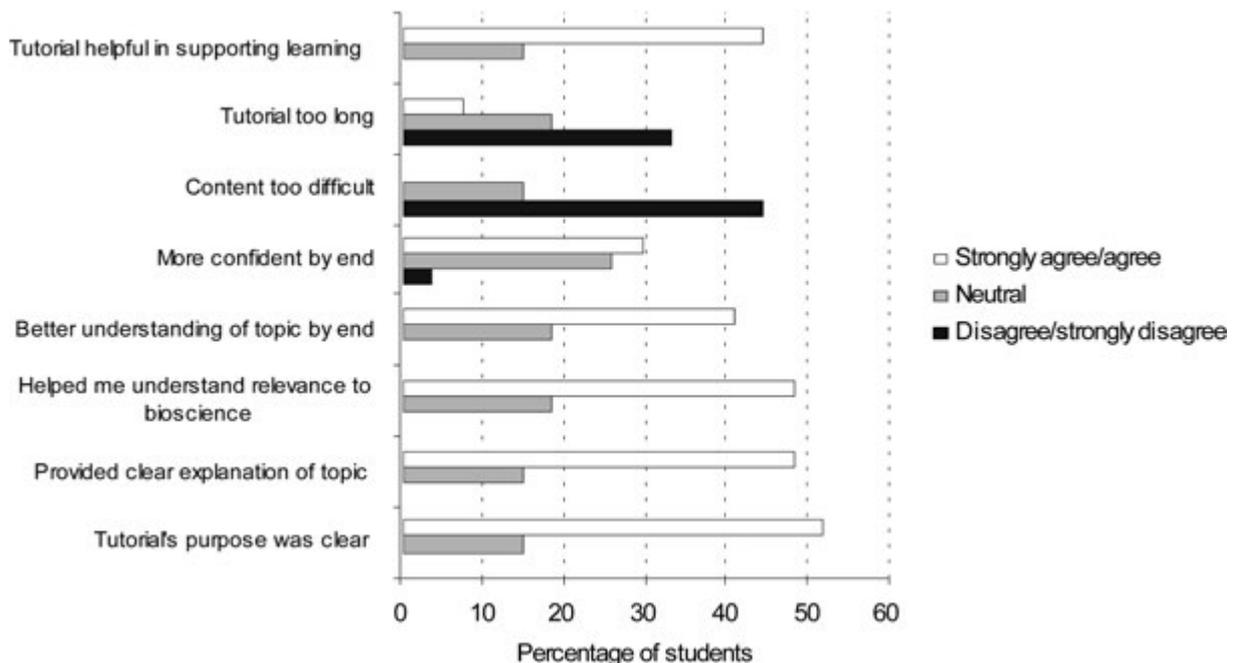


Figure 2: Summary of students' reactions to the 'Powers' tutorial

Overall, students liked the content of the tutorial, felt they learned from the tutorial and felt more confident with both the biological and mathematical topics after viewing the tutorial. Students' views regarding the remaining tutorials were similar. Tutors were also positive about the tutorials and agreed with students that the purpose of each tutorial was clear, as were the explanations and overall, the tutorials were helpful in supporting students' learning.

Some suggestions for improving the tutorials were made by both tutors and their students, e.g. incorporating graphical representations and animation to give the tutorials a more contemporary feel.

### **Students' attitudes towards mathematics**

The students who participated in this study were self-selecting, and therefore probably less maths anxious than many. Most students within this sample liked maths and felt that they were, to some extent, competent and confident in maths. However, almost 90% of students either strongly agreed or agreed that with more help and practice they'd be better at maths. As the student quote below illustrates, *biomathtutor* provides the opportunity for students to obtain help and practice in the maths topics it covers:

*"Builds up interest and motivation to work hard with maths. Biomathtutor helps students to understand the importance of maths to bioscience, and makes things simple."* (student)

### **Biomathtutor and curricula**

Most tutors felt there was potential for integrating *biomathtutor* into the curriculum and that they would like to use *biomathtutor*, in its entirety or in part, in classroom sessions. However, some envisaged encountering problems when trying to integrate this resource into their curriculum:

*"If it's support material I don't think there would be a problem... Things like the module specifications are so tightly drawn these days that if you want to put an assessment in that wasn't already described, then you would have to change the paperwork."* (tutor)

### **Conclusions**

The small numbers of students and tutors participating in this study, all of whom were self-selecting, limit the generalizability of its conclusions. Nevertheless, the following was apparent from the results obtained:

- Students' and their tutors' reactions towards the *biomathtutor* prototype learning resources were very positive. However, both groups proposed modifications regarding the design and structure of *biomathtutor*, with a view to giving it a more contemporary feel and enabling it to be used more readily by all learners, including those with specific disabilities.
- Students and tutors felt that *biomathtutor* had the potential to help students increase their understanding of maths and to see the relevance of maths to biology.
- Most students reported that using *biomathtutor* increased their competence and confidence in maths and that the content was relevant to their current studies. However, many of this self-selecting group of students felt they were already reasonably confident and competent in maths.
- Tutors agreed that *biomathtutor* would enhance students' knowledge in both biology and maths, felt the content was relevant to their students' learning needs, and that students would be able to integrate the new knowledge they gained into their current studies.

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# Mathematics Teaching Development through research in practice

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## Abstract

This paper is about developing the teaching of mathematics through research into teaching practice. It distinguishes between insider and outside researchers and focuses on an inquiry approach to teaching in which a teaching cycle of plan→act→reflect→feedback combines with an inquiry cycle of plan→observe→analyse→feedback to offer a research approach to exploring the teaching process in a way that leads to knowledge and improvement. The paper goes on to address the nature of research in teaching in terms of data collection and analysis and fruitful collaboration between insider and outsider researchers. It concludes with references to relevant theoretical perspectives that underpin the developmental research processes discussed.

## Research in mathematics learning and teaching

It seems to me that, in the field of mathematics education, there are two main goals for research. The first is to enhance *knowledge* in the field: knowledge about mathematics, about learning mathematics, about teaching mathematics, and about doing research in learning mathematics and teaching mathematics. The second is to enhance *practice* in the field: to enable *better* learning and teaching of mathematics, *better* researching, and so on. In association with these goals I will refer to two sorts of research, *insider* and *outsider* research. Outsider research involves study of the practices of other practitioners – from the *outside*; developing knowledge of practice. Insider research involves a study of one's *own* practices – from the *inside*; developing knowledge *in* practice.

Table 1 shows how these positions are related to each other.

Research	Enhancing knowledge	Enhancing practice
Insider	Know more about own teaching practices	Be more aware of teaching as we teach – make better informed decisions
Outsider	Know more about teaching practices generally	Support others in developing teaching – be more aware in our own teaching

Table 1: Knowledge, practice and modes of research

In this paper I focus on research into mathematics teaching, and consider how mathematics teaching can develop through research. I might ask, *why* and *how*?

*Why is research into mathematics teaching important?*

*How can teachers engage in research?*

## Teachers talking about teaching

In order to start addressing these questions, I will offer two quotations from a research study (outsider research) conducted some years ago in which three mathematics educators studied tutorial teaching of mathematics at first year university level with six university mathematicians who were the teachers in the tutorials [1, 2]. In each case we hear a teacher talking about teaching, taken from interview material following the observation of a tutorial.

### Teacher 1

I still just don't know how to teach it because a lot of this Group Theory course is going to be manipulation of symbols and silly little tricks. And I know, I understand this. I mean the fact that you can conjugate things and stick a sigma on one side and a sigma inverse on the other makes a lot of sense to me but they still haven't even changed bases on a matrix in, you know, in any course and, and that's so absolutely fundamental and it's going to underlie so much of what they do.

You know, really I should just take an hour and explain that to them. Um, 'cause, you know, I inevitably can't explain it very well in five minutes. All I can do is try and convince them it's the natural thing to do and that they shouldn't worry about it and that they will see it a lot. Just be happy with it.

### Teacher 2

Um, what I think I wanted to do there, was actually to, to interpret what, for example, the first part was:  $n^2 a^n$  tends to zero. So I wanted them to have the idea that what this is telling us is that  $a^n$  tends to zero faster than whatever  $n^2$  does. Which is a sort of, you grow a sort of feel to what – there's some sort of feel to it – and then actually prove it using the  $\epsilon$ - $\delta$ , so that I wasn't just jumping straight in with what I know; I wanted to, but actually, there's a reason for it.

I observe here that both teachers refer to "they" and "them" – they are talking about their students. They refer also to the mathematics on which they are working with the students. There are concepts they want students to know and understand; they indicate certain difficulties that they expect students to experience and refer to how they might address such difficulties. In the first case, this involves telling the students, or explaining; in the second it involves enabling students to get a "feel" for what is involved before going into the formalities of proof. These are pedagogic considerations: in what ways can we work with students to enable them to appreciate the mathematical concepts we have responsibility to teach. In the research project we noted different levels of pedagogic engagement in episodes we analysed.

Here, I am concerned with pedagogy. As teachers we can all recognise difficulties that students have with mathematics, and the literature alerts us to such difficulties and analyses many of them. A key question in pedagogic research concerns the approaches we take to addressing difficulties and enabling students to know and understand mathematics. We are aware that many students dislike, drop out or fail in mathematics. This suggests, perhaps, that the approaches to teaching that they have experienced are not doing a good job in helping these students to know and understand. How can we address these problems? How can we develop our teaching? I will suggest here that one approach is through research.

## The Process of Teaching

Initially, we might see the teacher as central to a process of mediating between the students and the mathematics (Figure 1a). Before this can happen, some kind of planning or preparation has to take place. What mathematics is to be learned? What approaches will be used to enable this learning? We might regard this as the

planning or design stage in the teaching process. Here the teacher has to think about what is involved for the students in approaching the particular mathematical concepts to be learned (Figure 1b).

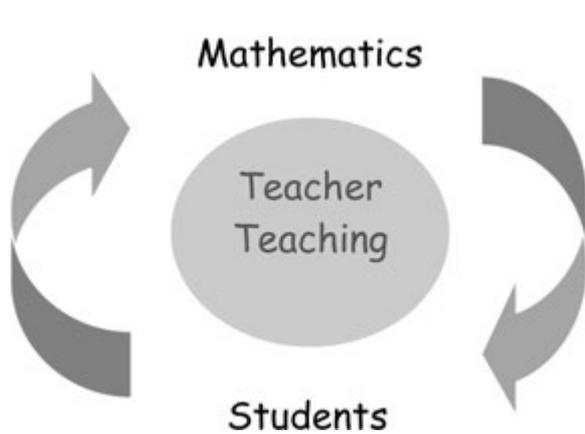


Figure 1a: Teaching links the students and the mathematics

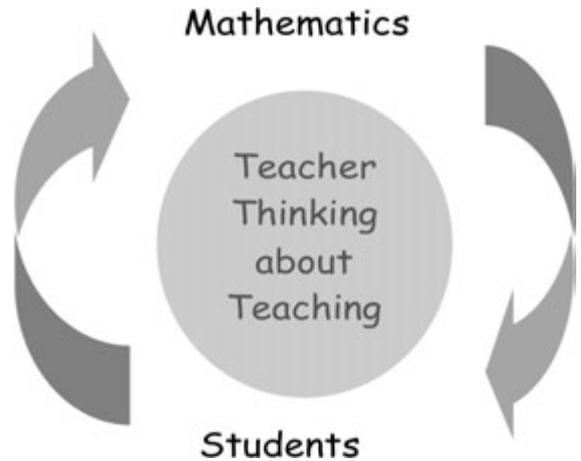


Figure 1b: Teacher thinks about how to make the links

The planning/design process, the act of teaching and subsequent reflection and feedback to future planning might be seen to form a teaching cycle as in Figure 2a. After any act of teaching it is usual to reflect on its outcomes – what responses there have been from students, whether particular difficulties have been observed, whether the teaching approach seems to have been effective. The teaching cycle can be seen to mediate between students and mathematics (Figure 2b).

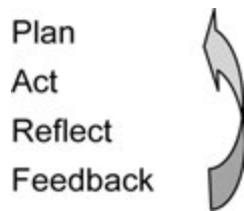


Figure 2a: A teaching cycle

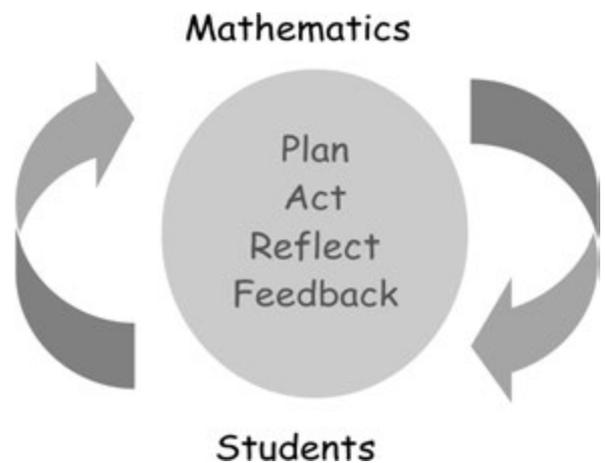


Figure 2b: A teaching cycle linking student and mathematics

### Inquiry in Teaching

In order to judge whether something is effective, we have to make judgments, and these can be better informed if we are clear as to their basis. To enable such clarity, we might extend the teaching cycle to an *inquiry* cycle (Figure 3a). If we overtly take notice of, or *observe*, what happens during the teaching process with students we can later not just reflect (think back to what happened) but actually *analyse* our observations. Again the cycle mediates between the student and mathematics (Figure 3b).



Figure 3a: An inquiry cycle

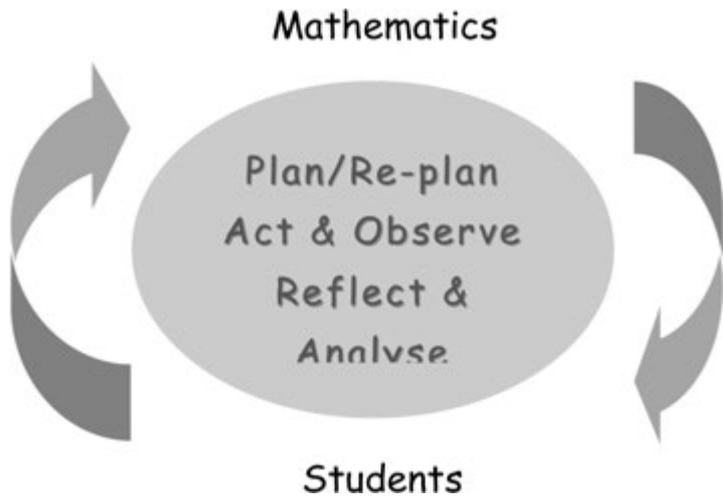


Figure 3b: An inquiry cycle linking student and mathematics

In order to observe and analyse, we need something more concrete than just our memory of the event. Memory is unreliable, and may involve interpretations that would not stand up in more systematic analysis. In order to engage in such analysis, we need some forms of data to analyse. We might split the inquiry cycle as follows, to emphasise the “inquiry” steps of *observe* and *analyse* (Figure 4).

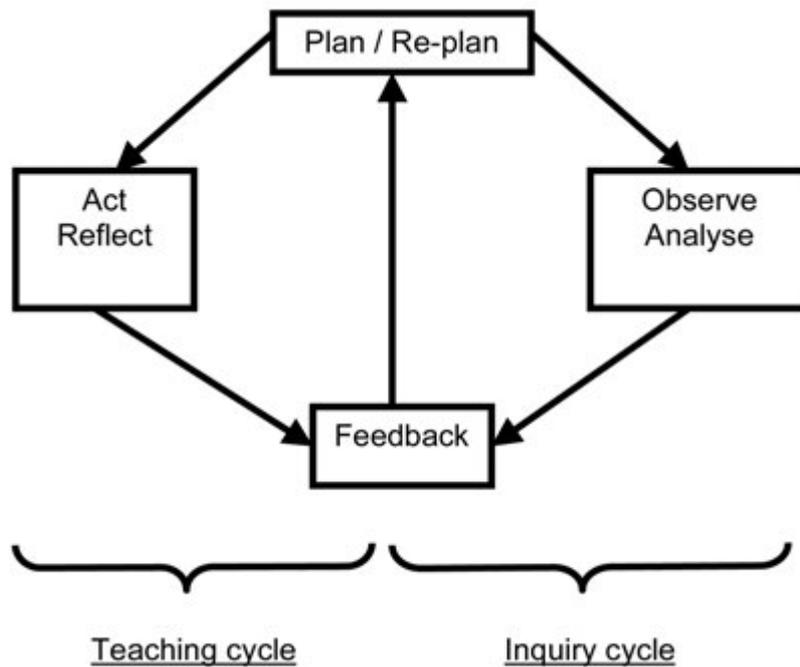


Figure 4: The two sides of an inquiry cycle in teaching

*Observe* means look at what we do as we do it (self-observation), and we can do this in a number of ways such as:

- keeping notes;
  - audio recording;
  - video recording;
  - other observer (keeping notes).
- } Here we collect data

Analyse means make sense of what happened in relation to what we did and how this related to our aims for the action. We can do this by

- reviewing notes;
  - listening to audio;
  - viewing video;
  - discussing with the other observer.
- } Here we analyse data

Analysis involves processes of questioning, organizing, categorizing and rationalizing. It allows us to suggest how plans might be reformulated according to the outcomes of analysis.

### Research in Teaching

When we talk about observation and analysis it sounds as if we are involved in a research process. Laurence Stenhouse [3, p. 126] has defined research as “systematic inquiry made public”. The inquiry cycle certainly involves systematic inquiry, but what about the extra tenet – to make public? It seems important that we keep records from analysis in order to justify the outcomes we claim, either to convince others or to publish our findings. With this extra step – keeping records to enable communication – we have a research cycle.

### Action Research and Developmental Research

The research cycle in Figure 5 looks somewhat like an *action research cycle*.

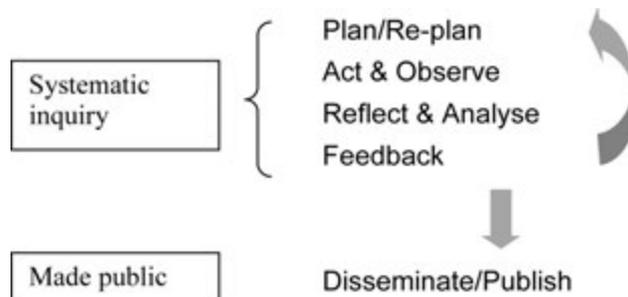


Figure 5: A research cycle that includes a teaching cycle

Action research might be defined as

‘the study of a social situation with a view to improving the quality of action within it’. It aims to feed practical judgement in concrete situations, and the validity of the ‘theories’ or hypotheses it generates depends not so much on ‘scientific’ tests of truth, as on the usefulness in helping people to act more intelligently and skilfully .... In action research ‘theories’ are not validated independently and then applied to practice. They are validated through practice. [4, p. 69]

A typical action research cycle might be as in Figure 6.

1. Declare a specific question/objective
  2. Plan the teaching with a focus on the question/objective
  3. Collect data with respect to the question/objective
  4. Analyse data with respect to the question/objective
  5. Report findings
- } The action cycle

Figure 6: An action research cycle

A difference between the research cycle presented in Figure 5 and the action research cycle (Figure 6) is that the former includes the action in teaching. So, the cycle in Figure 5 is not just a research cycle, it also includes, simultaneously, a teaching cycle. This emphasises the importance of teaching and research taking place *together*, with research informing teaching as part of a joint process. Such research can be referred to as *developmental research*. It influences development in the teaching process while charting this process.

## Planning for teaching and for research in teaching

### Planning for teaching

I come back now to the teaching cycle (Figure 2a), to look particularly at the nature of *planning*. What is it, as teachers, that we plan? I suggest we plan in three areas: content, didactics and pedagogy, possibly asking the following kinds of questions:

- Planning the content;
  - » what mathematics?
- Planning the didactics;
  - » how can the mathematics become accessible for students?
  - » what tasks, explanations, activity?
- Planning the pedagogy;
  - » what kinds of activity?
  - » how are the following used: lecture/presentation, group work, inquiry tasks, computer lab, personal study?

Didactics involves converting *mathematics* into *mathematics for learning*: a process of designing mathematical tasks in which students will engage in order to learn the designated content. Pedagogy involves a process of *designing the kinds of activity in which mathematical tasks are located*, such as lectures, group work, use of technology and so on. The teacher, in mediating between the mathematics and the students, is responsible for these layers of planning (Figure 7).

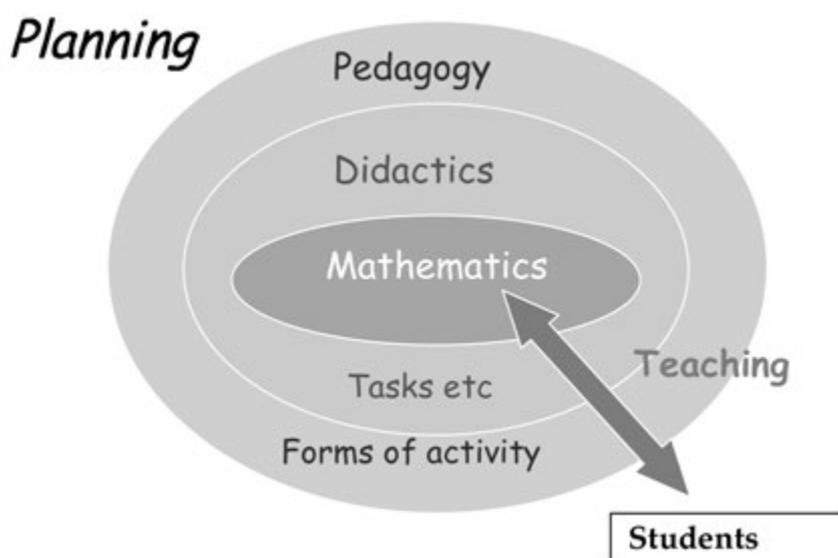


Figure 7: Teaching: involving a focus on mathematics, didactics and pedagogy

The layers are nested in the figure to indicate that mathematics is central to the entire process, that didactics works directly on the mathematics, and pedagogy on the didactics. Through didactics and pedagogy, mathematics is converted into activities which engage students in ways that enable them to learn mathematics with understanding. The teacher is responsible for the whole of this planning process, designed to mediate between the mathematics and the students.

### Planning for research

Planning for research parallels planning for teaching, guided by questions such as

- What is the focus of study?
- What will be observed?
  - » What kinds of data could be collected and analysed?
- What kinds of findings/knowledge are sought?
  - » What outcomes from the study are possible?

For example, the focus of the study might be on the mathematical topic, perhaps asking questions about what students find difficult about the topic, or it might be about the nature of tasks that allow students to really get to the mathematics and understand it. It might be on forms of questions that get students involved, or on the use of certain technology or software, such as the use of voting systems to engage students, or of a package such as *Derive* as a tool in learning algebra. Table 2 contains two research questions, focusing on the topic of *limits*, and relating to associated data and desired outcomes.

Research question	Associated data & analysis	Desired outcomes
What are the particular difficulties students have with <i>limits</i> ?	Students' responses to carefully designed questions on <i>limits</i> with a detailed analysis of their approaches and errors.	Clearer understanding of students' problems with <i>limits</i> to inform future design of tasks.
What kinds of questions relating to <i>limits</i> can be asked when using voting systems to engage students conceptually?	Observation of the responses given to different kinds of questions on <i>limits</i> with analysis relating to the quality of student engagement and understanding.	Clearer understanding of the possibilities of voting systems to engage students and of the particular kinds of questions that seem to be effective.

Table 2: Research questions, data and outcomes

The design of the research would vary depending on insider or outsider mode. For example, if the teacher is researching his or her own practice (insider research), he/she might deliberate as follows:

How can I use questioning effectively to challenge students and really get them thinking? I am interested in knowing more about different kinds of questions that I ask, or can ask; when to use different kinds of questions; what I am trying to achieve with my questions; what it means for questions to be used *effectively*.

The design of the research here must allow the teacher-researcher to collect data from teaching sessions, perhaps in audio or video form, or by organising for a colleague or research student to observe and keep notes, perhaps according to a pre-designed observation format. Analysis would then involve studying the data in relation to the goals for teaching and for students' learning.

Such data collection and analysis is designed to inform the teacher and provide clearer understanding of the learning and teaching process that can lead to more knowledgeable planning in the future. It can also provide more generalized knowledge for informing a wider population through a conference presentation or written publication. For example, a study of questioning could add to what is known more generally about forms of questioning, ways in which questions can be used, how students respond to different kinds of questions, or how teachers can use questions to achieved desired learning outcomes for students.

In outsider mode, research questions would be asked by a researcher studying aspects of learning or teaching as an external observer, perhaps in collaboration with the teacher. A collaborative mode can be particularly powerful, enabling the teacher to concentrate on teaching rather than trying to juggle the demands of both teaching and research at the same time, and providing the outsider with access to the teaching cycle from an insider's perspective. Figure 8 illustrates this schematically.

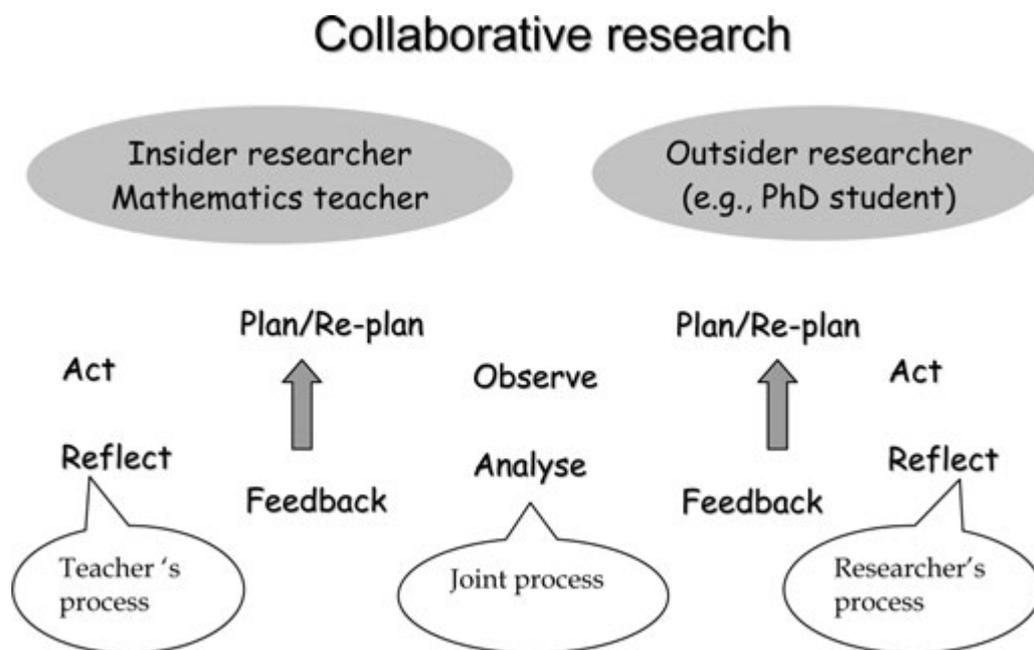


Figure 8: Collaborative research between insider(s) and outsider(s)

For the teacher (insider), *act* and *reflect* are part of the teaching cycle, planning for teaching. For the outsider researcher, *act* and *reflect* are part of the research cycle, planning for research. Both insider and outsider are engaged in *observation* and *analysis* albeit from their particular perspectives which are mutually informing. When insiders and outsiders collaborate in such ways, they can be considered as forming a community of inquiry.

### Community of inquiry

Inquiry communities are a special form of communities of practice. According to Wells [5], they are distinguished by forms of 'metaknowing' that develop from inquiry in reflective and reflexive processes. A community of inquiry is a *community of practice* (as theorized by Wenger [6]), in which participants take an *inquiry stance*, (a concept introduced by Cochran Smith and Lytle [7]). That is, they position themselves within the normal aspects of the practice in which they engage, as inquirers. In doing so, while engaging in normal practice, they also question and explore aspects of that practice, perhaps also trying out new approaches and evaluating them against the norms of the practice. In learning and teaching mathematics in university environments, the practices include lectures, tutorials, small group sessions, learning support, and so on. Our participation in a community of inquiry involves

critical questioning of the practices in which we engage in order to develop a more explicit and practice-related understanding of what we are doing and trying to achieve [8]. This is the 'metaknowing' of which Wells speaks.

Such ways of conceptualizing inquiry and inquiry-related practice draw on important areas of scholarship, some of which I will mention just briefly to suggest theoretical antecedents in areas of knowledge, reflection and action.

### **Knowledge, reflection, action**

In 1958 Polanyi [9] introduced the concept of *tacit* knowing to recognize the knowledge that professional practitioners bring to their practice without necessarily articulating this knowledge. Schön ([10] extends tacit knowledge to consider *knowledge-in-practice*, which, through *reflection-on-practice*, can become more overt, leading to *reflection-in-practice* which brings with it greater power over classroom decision making. Thus, practitioners become more aware of the knowledge they bring to practice through overt reflection on aspects of their practice.

Dewey wrote about reflection as 'demand for the solution of a perplexity is the steadying and guiding factor in the entire process of reflection' [11, p. 14]. So reflection offers an opportunity to attend to issues and problems in practice and work towards solving problems. Kemmis sees the reflective process as demanding action:

We are inclined to think of reflection as something quiet and personal. My argument here is that reflection is action-oriented, social and political. Its product is praxis (informed, committed action) the most eloquent and socially significant form of human action. [12. p. 141]

In other words, reflection is not just a contemplative process, it is an active ingredient in reviewing practice critically and suggesting forms of action that can lead to development and improvement. Mason [13] has proposed what he calls a *discipline of noticing*, in which awareness of what we notice in the moment of noticing is empowering in its potential for changing action. This accords strongly with Schön's notion of *reflection in action*. Through reflecting *on* action, practitioners become aware of aspects of their practice in ways that enable them to notice what is happening *while it is happening*, not just in retrospect. This *noticing in the moment* offers the possibility to change action wittingly during the action. 'Metaknowing' – 'reflecting on what we know' – implies reflecting on action in the process of engagement in action. Thus action in practice becomes more knowledgeable and open to development and change.

### **Research in/as practice**

Inquiry – engagement in research into practice – thus becomes part of the practice in which we engage. Collaboration between insider and outsider researchers is a central part of such practice. Although, in Figure 8, I suggested a separation between the activities of the teacher and the researcher, over time these distinctions become blurred as both partners take on aspects of the two roles. Chaiklin has written:

Social science research has the potential to illuminate and clarify the practices we are studying *as well as the possibility to be incorporated into the very practices being investigated*. [14 p. 394. Emphasis added.]

I suggest that working according to these theoretical principles in practice (which includes both teaching and research) allows us to work towards a science of teaching – teaching which is *less intuitive and more informed*.

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# Mathematicians' uses of Computer Algebra Systems in mathematics teaching in the UK, US, and Hungary

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## Abstract

The use of Computer Algebra Systems (CAS) is becoming increasingly important and widespread in mathematics research and teaching. In this paper, a questionnaire study enquiring about mathematicians' use of CAS in mathematics teaching in three countries, the United Kingdom, the United States, and Hungary, is reported. Specifically, the study attempts to examine the current extent of CAS use in universities, describes some CAS-assisted teaching practices of mathematicians, suggests factors that influence technology integration into university level teaching, and highlight mathematicians' views on the role of CAS in mathematical literacy. It is also shown that there are considerably fewer differences in mathematicians' views on the use of CAS in teaching between the participating countries than school-level studies suggest on teachers' various beliefs. Finally, the study highlights the importance of collaboration between mathematicians and educational researchers to be able to enhance technology in mathematics teaching and learning.

## Introduction

Anecdotal evidence suggests that Computer Algebra Systems (CAS) are increasingly used in the teaching and learning of mathematics at the university level. However, little is known about the current extent of CAS use in universities, the various practices mathematicians employ in their teaching with technology, and mathematicians' views on the role of CAS in the mathematical curriculum and literacy of students. Notwithstanding studies and descriptions of innovative practices in various research and practitioner journals [1], there appears to be no evidence of comprehensive studies that broadly examine technology use in university classrooms. To bridge this gap, a two-phase international study was designed that aims to answer several questions.

- Firstly, to what extent and manner are CAS currently used in university mathematics departments? At the school level, a number of large-scale studies have been conducted either nationally [e.g. 2] or internationally [e.g. 3] which provide benchmarks for other studies and allow the documentation of changes occurring in this area over time. This study aims to provide a partial overview of the current use of CAS in universities.
- Secondly, what mathematical and pedagogic beliefs and conceptions do mathematicians hold with regard to CAS, and how do they envision the role of CAS in university-level mathematics education? School level studies suggest that teachers' conceptions, beliefs, attitudes, and motivations (of mathematics, mathematics teaching, and technology) are important factors influencing the integration of technology [4]. Thus, it is important to examine these factors at the university level as well.
- Thirdly, to what extent do nationally situated teaching traditions, frequently based on unarticulated assumptions, influence mathematicians' conceptions of, and motivation for, using CAS? School level

studies also indicate that social and cultural factors play an important role in integrating technology into mathematics teaching and learning [5]. Therefore, the design of the study follows an international comparative approach to be able to account for cultural influences as well.

## Methods

The study design followed a two-phase mixed methods approach [6]. The first, qualitative, phase of the study, comprised interviews, class observations, and a review of curriculum materials of 22 mathematicians in Hungary (HU), the United Kingdom (UK), and the United States (US). Findings of this phase [7] were incorporated into the development of an on-line questionnaire.

The second, quantitative, phase comprised a questionnaire which was sent to 4,500 mathematicians in the participating countries. The questionnaire included sections dealing with mathematicians' personal and academic characteristics; mathematicians' views on the role of CAS in mathematical literacy, teaching and learning; the factors hindering CAS integration into teaching; the resources available for CAS-assisted teaching; the description of current CAS use; and, space for optional written responses. The analysis of 1103 responses attempts to provide answers for the posed research questions.

## Results

### High response rate

Overall, 1103 mathematicians responded to the questionnaire (average 25% response rate, by country: US-20%, UK-25%, and HU-46%), which constitutes a surprisingly high response rate according to the web-survey literature. In addition to responses to closed questionnaire items, mathematicians wrote an approximate total of 150 pages for the optional open questions and sent approximately 600 e-mails, many of which included relevant comments. Furthermore, 297 mathematicians volunteered to participate in future technology-related studies.

The high response rate and the generally positive feedback suggest that mathematicians are interested in learning about technology applications in mathematics teaching and many of them are open to discuss educational issues. In addition, changes in higher education during the past decade such as the increased enrolment in universities, the lower student interest in Science, Technology, Engineering, and Mathematics subjects, and difficulties in school-level education, have resulted in a decline in mathematical preparedness of students entering universities. Furthermore, the emergence of new technologies available for teaching opened new perspectives and intensified demands for the changes in teaching practices. The observed weaknesses in students' mathematical preparedness and the availability of technology prompted numerous mathematicians to experiment with innovative teaching practices and to question their more traditional pedagogy. Moreover, in many cases the integration of technology into undergraduate teaching is seen as a way to revitalize teaching and to assist students in raising their level of mathematical understanding. Although university-level mathematics teaching is undergoing considerable changes and is in need of assistance, little attention has been paid to teaching issues at this level by the educational research community. In particular, little is known about the current extent of technology use and mathematicians' practices in university teaching [7]. These new challenges and the increased openness and interest of mathematicians and mathematics education researchers could offer a good opportunity for collaboration to resolve problems in mathematics teaching and learning.

### Extensive use of CAS in research

Table 1 shows that approximately 67% of participants indicated that they use CAS for their own research on at least an occasional basis.

Frequency		Never (%)			Occasionally (%)			Frequently (%)		
Country	n	HU	US	UK	HU	US	UK	HU	US	UK
CAS in research	1089	33	34	33	34	34	34	32	32	32
CAS in teaching	920	42	42	53	42	41	38	15	17	9

Table 1: Mathematicians' use of CAS in research and teaching

This percentage is high even when allowing for the fact that mathematicians who have some kind of connection to CAS were strongly represented among those responding to the questionnaire. After accounting for this likely bias it might be inferred that every third, or more likely, every second mathematician uses CAS in their own research. Thus, there are a large number of mathematicians who have acquired strong working knowledge of at least one mathematical software package and this knowledge can be readily utilized for CAS-assisted teaching. In fact, in the Structural Equation models developed for modelling the influences of CAS integration into teaching the variable *CAS-use-in-research* is the strongest predictor of the variable *CAS-use-in-teaching*, explaining 27% of the variance in the model. Proficiency in the use of a software package offers an advantage to mathematicians over teachers as they often do not require initial training for software before beginning to use it in their teaching.

### Extensive use of CAS in teaching

Based on Table 1, 55% of the participating mathematicians reported that they utilize CAS for their teaching on at least an occasional basis. Similar to the research use of CAS, a possible bias should be considered in the sample, which might lower the percentage of CAS use in teaching in the overall population. However, there are other kinds of technologies used in university-level mathematics teaching apart from CAS. Therefore it is not unreasonable to assume that more than one third, or even one half of mathematicians use technology in their teaching. Comparing this result to school level studies it can be inferred that technology use at universities is substantially higher than the 5-10% level reported at the school level [8]. This result also indicates that mathematicians have already accumulated an immense expertise and knowledge in technology-assisted teaching, although it is only sparsely documented.

Due to the high level of technology use in their teaching, mathematicians have likely developed significant innovations due to their greater freedom in developing their own curriculum and teaching practices. Unlike teachers, mathematicians are often not so tightly constrained to follow pre-existing curricula and are often free to experiment with innovations. Therefore, together with their considerable knowledge of mathematics and software there may be striking innovations already employed at the university level, which can be utilized not only at other universities, but also at the school level.

### Purposes of CAS use

Mathematicians highlighted some practices in their written responses in relation to the purpose of CAS use in their teaching. Mathematicians mentioned that they use CAS to

1. encourage group work in classes;
2. visualize and project images;
3. assist experimentation, exploration, and discovery in classes;
4. offer realistic, complex, or real word problems for students;
5. devote more time for conceptual problems;
6. motivate students in classes;
7. prepare and offer homework assignments; and,

8. check solutions of student assignments and worksheets.

The frequency of the appearances of these themes in the written responses was widely varied. While visualization and experimentation were mentioned more than 100 times in the written responses, motivational issues only appeared less than ten times. Certainly, frequencies are not the most precise indicators of the importance of particular themes, but these can imply what mathematicians consider important with regard to CAS integration in mathematics teaching.

The most frequently mentioned themes in mathematicians' responses were visualization of mathematical concepts:

*"I think it is useful to be able to visualize certain things that would be hard to illustrate on the board." [case: 490, US]*

and enabling students to engage in experimentation and discovery which the powerful computational feature of software allows:

*"I hope to encourage students to experiment more widely than they would be able to if doing calculations by hand." [case: 158, US].*

In addition, discovery activities can create an environment where students learn mathematics in a manner akin to undertaking mathematical research:

*"Exploring is like doing research. And that is what we are supposed to be teaching. CAS allows exploration without the tedium." [case: 395, US].*

The limitation in length of this paper prevents further discussion of the current use of CAS, but detailed analysis will be forthcoming in other publications. Furthermore, close examination of CAS-enhanced teaching activities should be carried out utilizing qualitative research approaches.

### **CAS role in mathematical literacy**

Mathematicians' views on the role of CAS in mathematical literacy were an important component of the questionnaire. Ten statements were listed in the questionnaire and mathematicians were asked to rate these statements according to their agreement or disagreement on a five-point Likert scale from Strongly Disagree (1) to Strongly Agree (5). Statements were categorised into three groups:

1. CAS as part of the curriculum—including statements about the importance of students gaining CAS knowledge during their studies and whether the knowledge of CAS is beneficial for their future studies and career;
2. CAS influence on mathematical research—including statements about whether or not CAS changes the way mathematical research is done; and,
3. CAS influence on the curriculum—enquiring about whether or not CAS imposes changes in the university mathematics curriculum.

Responses indicated that mathematicians view the role of technology, particularly CAS, positively in terms of mathematical literacy and in university curricula. They agreed (average responses exceeded 3.5 score) that proficiency in CAS use is beneficial for students' future studies and career, and they suggested that CAS will eventually become an integral part of undergraduate mathematics curricula.

Participants explained that CAS has become a genuine mathematical tool for mathematicians: "It's an important tool for a working mathematician." [case: 441, US]

and many mathematicians use it for their research as it is a part of modern mathematics: "I use it in my research, and like it, so why not? It's a part of modern maths." [case 789, UK]. Therefore, students should have knowledge

of this new tool: "I use it in research, shouldn't the students? [case: 173, US] and even some mathematicians stated that it is inconceivable to teach without technology these days: "the computer is a major tool throughout much of mathematics, pure and applied. It is simply inconceivable (to me) that we would train mathematicians who cannot use it." [case: 957, UK] and in many areas of study and numerous professions the knowledge of using mathematical software is essential: "They need to learn that these things exist because they may be using them in their careers." [case: 608, US]. There was a general agreement among mathematicians about the future integration of CAS into mathematics curricula, but the extent of such integration needs to be debated within the mathematics community.

The statistical analysis of the data indicated little difference among the three countries in mathematicians' perspectives on the role of CAS in mathematical literacy and curricula. This finding reinforces results in the first phase of the study suggesting that mathematicians are more internationally mobile and more internationally aware than teachers and that their thinking and beliefs are less culturally-based than those of teachers. The study revealed that there is a high percentage of foreign-born mathematicians working in UK and US mathematics departments and numerous Hungarian mathematicians have studied and worked outside of Hungary at some point in their career. This fact could contribute to the slight differences measured among the participating countries.

## Discussion

Initial results of this study indicate that mathematicians use technology for teaching more extensively than school teachers. Numerous mathematicians have accumulated extensive knowledge about mathematical software packages through their own research. Coupling this knowledge with their expertise in mathematics as well as with the freedom of developing their own curriculum materials provides a rich opportunity for innovations in technology-assisted teaching. In addition, mathematicians look favourably on the role of technology in mathematical literacy and curricula. Therefore, it is likely that there are significant innovations and successful teaching practices already existing at the university level. Consequently, it would be advisable to pay closer attention to mathematicians' technology-assisted teaching such as documenting and researching these practices and innovations. This could significantly contribute to not only advancement in research and practice at universities, but also, by extension, at the school level.

It can be observed that during the past two decades technology-related studies focused almost entirely on the school level. Certainly, there have been new research groups and Special Interest Groups founded to deal with issues in higher education and technology is part of their research scope. However, it would be advantageous for educational researchers to focus more attention on university level research. In particular, mathematicians are becoming more open and attentive to educational research and so opportunities are becoming increasingly available for such research at this level. Also, this situation creates a good opportunity for mathematicians and mathematics educators to jointly engage in discussions about teaching issues at the university level, and this will likely contribute to enhancing students' learning at all levels of education.

This study has provided an overview of general use of CAS at the university level, but it was not possible to gain thorough details about what is happening in mathematics departments nor to adequately account for how mathematicians use CAS in their teaching. Therefore, to continue examining CAS and various technology applications and innovations at universities, it would be desirable to closely examine and document particular technology-assisted teaching practices and teaching programmes with different levels of technology integration.

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# Roles of assessment in learning in statistics and mathematics

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## Abstract

In the nexus between principles and practice in tertiary assessment in statistics and mathematics, the variety and extent of demands and pressures on assessment packages can sometimes appear overwhelming and even contradictory. Amidst the balancing of formative, summative, flexible, continuous, rich and authentic assessment with demands for criteria and standards-referenced assessment, and developing generic graduate capabilities such as teamwork, problem-solving and communication skills, lurk the problems of over-assessment and the politics of pass rates and attrition. The many dimensions of the assessment challenge are complicated in introductory statistics and mathematics courses by the diversity of student cohorts in which the wide range of backgrounds, programmes, motivations and study skills need consideration in designing appropriate assessment and learning packages. This paper discusses issues, challenges and strategies in designing and implementing integrated assessment and learning packages in statistics and mathematics, particularly in early undergraduate years for both service and core courses. Comments on plagiarism, cooperative and group work are included. Examples are given in both statistics and mathematics, and similarities and contrasts in different assessment packages are highlighted.

## Introduction

Calls for tertiary educators to assess what they value, to identify learning objectives, and to align assessment with objectives, appear in both general and discipline-specific higher education literature emphasizing the role of assessment in learning. Although this sometimes causes tensions, it is important for disciplines to be proactive in analysing, developing and proclaiming the pedagogical aspects of their disciplines, including points of agreement or otherwise with general higher education literature and viewpoints. Assessment is often discussed as if it is an entity in itself, whereas assessment is, and should be, an integral part of a holistic approach to learning and teaching. Tertiary educators' complaints of previous eras that students learn only for assessment are fading with the growing understanding that assessment should be designed for, and aligned with, student learning. Naturally students learn for assessment. From their perception, what is assessed must be of value. 'Good' learning and assessment packages in introductory tertiary mathematics/statistics units are integrated, balanced, developmental, purposeful packages with well-structured facilitation of student learning across the cohort diversity.

After some brief comments on literature on assessment in general higher education, and in tertiary teaching of mathematics and statistics, including use of criteria and standards, this paper focuses on four examples illustrating aspects of assessment in different contexts. These examples are from one university but the contexts and characteristics of the student cohorts are analogous to many. The term 'unit' is used here to refer to any module, subject or course for which a student can enrol and receive a grade on completion. Each case includes a summary of the information about the unit and student cohort necessary for any discussion about assessment. The first example is an introductory data analysis unit that is both a service and a core unit, whose central theme is the planning, implementation, analysis and reporting of data investigations. Its learning and assessment

package is balanced over components of its objectives. The second example is an introductory probability and distributional modelling unit that is core for mathematics majors but also has enrolments from other degree programmes. It progresses through unpacking, analysing and extending concepts, knowledge and skills, with emphasis on linking with real contexts and data. Its learning and assessment package is built around learning problem-solving and modelling skills.

Both these units are first year units. A second year linear algebra unit that is also core for mathematics majors but with enrolments from other degree programmes, looks towards industry problems, applied research and computational mathematics. The assessment challenges in this unit are the balance of assignments and tests, and of theory, applications and computing. The fourth example is a second year engineering unit that consists of three components. Half of the unit is introductory data analysis as in the first year unit above, one quarter is introductory theory related to distributions, and the remaining quarter is an introduction to computational mathematics. Hence the challenges are very much of balance – balancing components, workload, objectives and development of skills and knowledge.

Each of the four units includes both group and individual assessment tasks. The collaborative tasks are oriented to particular objectives and are of a nature that calls for group work. A characteristic of good learning and assessment packages in mathematics and statistics is that assessment tasks that carry weight towards the overall result but that are due during semester, are formative *and* summative – these are referred to here as formative/summative. The feedback on such components of assessment is vital for student learning. End of semester components of assessment are summative only. Components of assessment that carry no weight are formative only – in mathematics and statistics such components are primarily for learning and only partially for self-assessment. The overall balance of assessment components and their weights is a key consideration in the gradual development of student skills and operational knowledge.

The combination of qualitative and quantitative information on student performance is important in evaluating the effects of components of assessment in the overall learning package for a unit. In all four examples discussed here, examination of data on student performance in components of assessment assists considerably in understanding strengths and weaknesses of the assessment schema and in the ongoing process of developing aligned assessment strategies and objectives.

## Objectives, criteria and standards

The following two quotes represent two important aspects of the roles of assessment in learning in general in higher education:

*If learning really matters most, then our assessment practices should help students develop .. skills, dispositions, and knowledge.....[1]*

*Students study more effectively when they know what they are working towards..... Students value assessment tasks they perceive to be 'real' [2]*

Such thoughts are reflected within a more discipline-specific approach in the statistics education literature with authors urging care and in-depth consideration of objectives, goals, contexts and content [3], [4]; emphasis on data, statistical literacy and reasoning [5]; and calls for statistics educators to assess what they value [6].

The above quotes and references all entail clear identification of objectives and explicit aligning of assessment with objectives [2], [7]. Aligning of assessment with objectives is an iterative and reflective, rather than sequential, process. Identification of valued assessment tasks helps in the identification of objectives and vice versa. The process involves identification of the purpose of the learning, of what the students are bringing to their learning, of how they learn and manage their learning, and of their perception of the roles of this particular learning in their courses and their futures. In mathematics and statistics, particularly at the introductory level, this must also

take account of a wide range of backgrounds, programmes, motivations and study skills. In principle, this process is not overly difficult, but recent pressures for staff in tertiary assessment can tend to obscure or complicate issues. These pressures include seeking balances and paths amongst considerations such as:

- formative, summative, flexible, continuous, rich and authentic concepts of assessment;
- generic graduate capabilities;
- work-integrated learning;
- criteria and standards-referenced assessment;
- higher education fads, generalisations across disciplines and arbitrary rules;

plus challenges such as:

- avoiding over-assessment;
- the politics of pass rates, attrition and standards;
- the increasing diversity of student cohorts;
- the desire of some for 'instant gratification';
- managing and controlling workloads of students and staff.

Perhaps it is not surprising that a report of a survey of US statistics educators, [5], comments that of all areas of statistics education, assessment practices have undergone the least reform.

Contrary to the fears of tertiary staff who have been exposed to only the verbal descriptors of criteria- and standards-referenced assessment, the messages in leading research in this area are consistent with what is regarded as desirable by staff and students in assessment in statistics and mathematics. Marks are certainly valid as measures of achievement against criteria. Whether letters, words or marks are used, the critical aspect is identifying what they represent. In criteria- and standards-referenced assessment, it is the *configuration* or *pattern of performance* [8] that enables standards referencing. The configuration comes about through a combination of the construct of formative and summative assessment (aligned with student learning across the spectrum appropriate for the purpose and cohort), and the construct of timing, types and weights of assessment tasks. In [8] the importance of *exemplars* (such as marked past student work, representative assessment tasks and model solutions) is emphasized in identifying the characteristics (or criteria) of each component of assessment, with verbal descriptors to draw attention to salient criteria at different points.

### **Example 1: Statistical Data Analysis 1**

This unit is core in all science or maths degree programmes, including education with maths as a teaching area. Approximately 600 students a year enrol over two semesters. The theme of the unit is basic statistical data concepts and tools and using them in real data investigations. The objectives cover two overlapping aspects: tools and building blocks of procedures, concepts and operational skills; and synthesis of choosing, using and interpreting statistical procedures in whole data investigations. The structure, examples and learning experiences are built around real data investigations from first ideas through to report. The content can be seen in chapters 1-8 of [9] and progresses from planning, collecting, handling and exploring data through to ANOVA and multiple (including polynomial) regression, with the procedures of the last two topics done only through a statistical package. The learning and assessment package (with assessment weights) consists of:

- computer-based practicals on datasets from past student projects (formative only);
- worksheets with full solutions (formative only);

- fortnightly quizzes of the fill-in-gaps and short response type, with the best 5 out of 6 contributing 10%;
- a work-folder containing the student's ongoing work on the worksheets and their marked (collected) quizzes, contributing up to 3% in proportion to work-folder content;
- a whole semester group project in planning, collecting, analysing and reporting in context of group choice (20%);
- an in-semester test, similar to quizzes 1-4 (10%);
- an end of semester exam, similar to quizzes, with more weight on the content of quizzes 5 and 6 (57%).

Various aspects of the own-choice project, its impact on learning and teaching, and examples from over 1500 students projects, are included in [10]-[13]. Research on numeracy and its connections with statistical reasoning for the cohorts in this unit can be found in [14], [15]. For a few years there was also an optional essay on how statistics revolutionised science in the 20th century, contributing 10% if the mark improved the student's overall result. This item was dropped because it almost never improved a student's result, and because it tended to attract, and distract, students who could least afford the time. It was an appealing assessment item for staff and some students, but, on the basis of data, we concluded that it was not a sufficiently valuable component of the overall package to warrant student or staff effort.

The own-choice group project teaches and assesses investigation and synthesis of procedure choice and interpretation. It is group because the task needs a group. The students receive feedback on their ideas, and assistance throughout the semester. They have access to past projects and model reports, and past datasets are used in class demonstrations and practicals. Three criteria are given with guidelines, descriptors and standards, and each group receives written comments and marks for the three criteria. These are:

- Identifying context and issues, planning and collecting of data, quality of data and discussion of context/problems;
- handling, processing, preparing and understanding data and issues; exploring and commenting on features of the data;
- using statistical tools for statistical analysis and interpretation of the data in the context/issues.

The students form their own groups, with help from staff as necessary. Projects are retained and are designated as 'published' material. Students may refer to past projects if they wish. They know that a copied project is given zero marks. In over 1500 projects, this has occurred fewer than 10 times. All members of a group sign a cover sheet and receive the same mark. Within groups, there are almost never uneven contributions in criteria (i), seldom in (ii), and within-group allocation of tasks assists in avoiding them in (ii) and (iii). Students who emerge as leaders in criteria (iii) tend to learn more, need less revision and do well overall.

For the first semester class of 2006 of which 260 students sat the end of semester exam, Figure 1 gives the plots of the total marks on the end of semester exam paper versus their project mark, and versus their total for the quizzes. The quizzes provide guided learning of tools, procedures and operational skills, and also provide exemplars for the exam. Both these contribute to the moderately strong positive relationship in exam mark versus quiz totals. The greater variation of exam marks amongst the lower quiz marks reflects the mix of abilities amongst those who tend not to do regular assignment work. Although the project helps develop procedural knowledge, it is designed to teach and assess more of the synthesis aspect of the objectives, so the low but slightly positive relationship with the exam marks is appropriate.

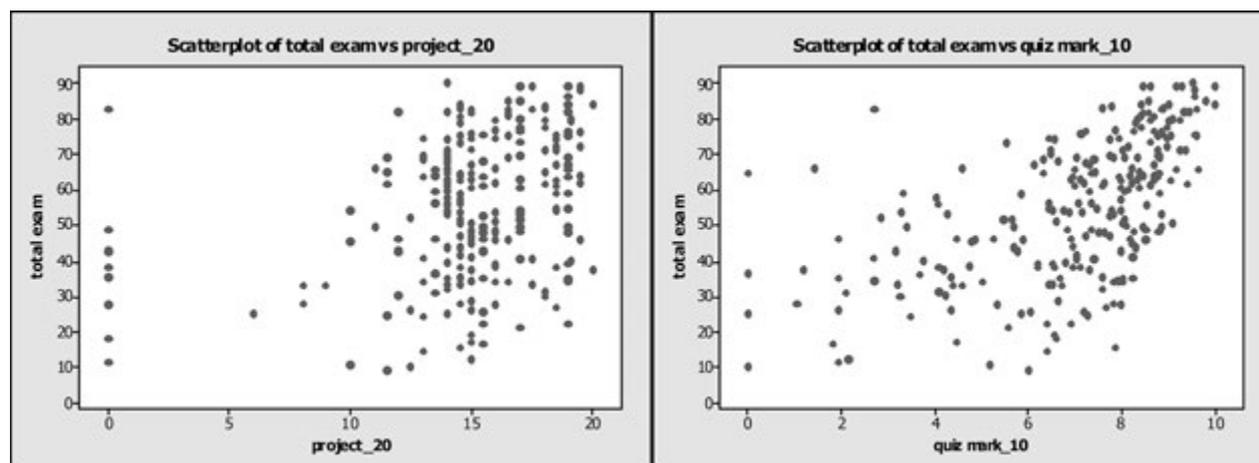


Figure 1: Plots of the total exam mark versus the project mark and the quiz mark in Example 1, Statistical Data Analysis 1, for semester 1, 2006

## Example 2: Statistical Modelling 1

This unit is core in all maths degree programmes, including education with maths as a teaching area. It is also taken as an elective by some science and other students. Approximately 120 students a year enrol. The unit builds skills and foundations in concepts and thinking in introductory probability, conditional arguments, distributional and initial stochastic modelling for applications to a wide range of areas, from communication systems and networks to traffic, biology and financial analysis. The unit analyses and extends prior understanding (and misunderstanding); links with data, observation and simulation; and uses, and hence consolidates, foundation calculus and algebra skills in new and different contexts. The whole approach is development of problem-solving and modelling skills.

The formative components of the learning and assessment package include:

- an initial general probability reasoning questionnaire (PRQ) to seed thought and discussion (introduced in 2004);
- class activities, simulations, selected computer modules, and worksheets with unlimited help;
- preliminary discussion points or exercises at the start of each topic to identify and unpack prior knowledge in order to analyse and extend. Development of these was completed in 2005.

More information on the unit and roles of the preliminaries can be found in [13].

The formative/summative and summative components described below, with their % contribution to the overall result given in brackets, are all oriented to problem-solving:

- four assignments based on examples and worksheets, with problems in authentic contexts (20% before 2006; 16% in 2006);
- a group project in which two everyday processes that could be Poisson are chosen in free choice, data collected, and Poisson-ness investigated through tests and graphs (10%);
- an end of semester exam involving problem-solving based on activities, worksheets and assignments, ranging from simple to more challenging in life-related contexts (70% before 2006; 66% in 2006);
- four special tutorials structured for immediate hands-on learning through group exercises, introduced in 2006 (8%).

Students design and bring their own summaries to the exam. For the four special tutorials, students are allocated to different groups for each and full collaboration is required, with groups ensuring that explanations are shared. Assistance is available and credit is for participation, with the group's attempt signed by all group members. Solutions are provided later and the groups' attempts are collected but not marked because the credit for this component is for participation. For more information on the group project see [12], [13], and on the new tutorial group exercises, see [16].

Introduction of the tutorial group exercises component was based on observation of the most and least successful learning styles of the students as part of the research and reflect component of the 'field' research cycle. It was decided that more active learning was needed to develop skills in discerning and applying relevant information, and transference to new situations. A number of topics were identified as most needful of immediate involvement of students in active problem-tackling in a problem-solving environment as described in [17], namely *an emotionally and cognitively supportive atmosphere where students feel safe to explore, comfortable with temporary confusion, belief in their ability and motivation to navigate stages*.

Tutors and students voted the experiment an outstanding success. The tutorials were buzzing and early departures were practically non-existent. Other tutorials also benefited significantly through increased attendance and participation. Student opinion was that four was the ideal number. As the assignments provide exemplars for the exam, it is not surprising that analysis, whether in terms of best subsets, forward or backward fitting, shows that they are most important in predicting exam scores. In both 2005 and 2006, in the full models for exam scores in terms of all other variables, only the assignments ( $p = 0.000$ ) and the PRQ score ( $p = 0.009$  in 2005,  $p = 0.032$  in 2006) were significant. In both years, the relationships of the assignment marks on the other non-exam components of assessment were then investigated. In 2005, significant predictors of the assignments score were the group project mark ( $p = 0.060$ ) and the PRQ score ( $p = 0.016$ ). In 2006 the tutorial group exercise contribution was the *only* significant predictor ( $p = 0.004$ ) of the assignments score. It must again be emphasized that the score from the tutorial group exercises is solely for participation, and that they are intended to develop learning and problem-tackling. Hence the above analysis provides further support for the qualitative evidence that the tutorial group exercises are fulfilling their intended roles as significant enablers for all students.

### **Example 3: second year linear algebra unit**

This unit is core in all maths degree programmes, and some physics programmes. It is also taken as an elective by some engineering students. Between 70 and 90 students a year enrol. The examples and learning experiences in the unit are motivated by higher level needs in mathematics generally, particularly computational maths, and by applications based on experience with industry problems. Although the students are all maths-oriented, there are differences within the cohort, for example between those doing a double degree in maths and business and those doing double degrees in maths and information technology or engineering. The challenges of balancing theory, applications and computing in linear algebra are thus augmented by the cohort diversity. To encourage student engagement and hence student learning, some changes were made in the continuous (that is, during semester) assessment between 2003 and 2005. A number of questions were raised, including: did these changes help or impede student learning, and does the computing help in the theoretical learning?

The assessment package in 2003 consisted of:

- three Maple group assignments totalling 21%;
- a mid-semester test worth 15%;
- a final exam worth 64%.

The lecturer's observations plus student feedback were that the Maple group assignments were too heavy for 7% each, and that, similarly to Example 2 above, students needed more structured help with their learning. In 2005, the assessment package was adjusted to:

- two Maple group assignments totalling 24%;
- three quiz-style assignments totalling 16%;
- a final exam worth 60%.

The final exams were similar in style, format and level in both years. They have both theory and applications but no Maple use. More details of the content and the assessment can be found in [18].

Analysis of data from the assessment components showed that, for both continuous assessment programmes, a Maple group assignment and a 2005 quiz or the 2003 test combined as best predictors of the exam [18]. This provides support of claims in the literature that both theory and computing contribute to overall learning and understanding in linear algebra, and reassurance that changes in the continuous assessment programme are not detrimental and do appear to assist in learning. The lecturer had concerns about the grading of the continuous assessment, but could now tackle this with confidence in the programme's facilitation of student learning across the theory and practice components of the unit.

#### **Example 4: a second year engineering maths unit**

This unit is core in all engineering programmes, with 350-500 enrolled in second year programmes. The unit was new in 2007 but composed of sections common across previous post-first year engineering maths units. The first half of the unit is as in Statistical Data Analysis 1, and as given in all engineering programmes since 1994. The second half is half introductory numerical analysis and half distributions, linear combinations of normals, and introduction to reliability. The level of the unit is first year work in Maths and Science programmes, but more compressed. Apart from the compression, its challenge for engineering students is that it is not straight calculus and algebra, and any calculus and algebra that are needed must be at students' fingertips in new contexts. Also, all the statistics is full of new concepts and new ways of thinking.

The focus in all mathematics and statistics units is learning by doing, but this is emphasized even more in this unit. The assessment package consists of:

- for the data analysis half, computer-based practicals on datasets from past student projects (formative only);
- worksheets with full solutions (formative only);
- five quizzes as assignments on the statistics parts, with quizzes being of the fill-in-the-gap or short response type (14%);
- whole semester group project in planning, collecting, analysing and reporting a data investigation in context of group choice (20%);
- numerical analysis assignment (6%);
- end of semester exam, ensuring overall coverage correctly proportioned (60%).

The statistics quizzes are designed for efficient and effective learning, and there is evidence of their value over years and units. The strategy was introduced in the late 1990's in a short Master of Business Administration (MBA) unit with a highly diverse cohort of students holding full-time jobs. It was then developed further in an engineering unit when the data analysis became a half-unit module. The aim was to decrease demand on student time while retaining the full data investigation project. An unexpected and interesting side effect in the engineering unit was a great reduction in copying. Students still worked together on the quizzes but argued

with, and explained to, each other instead of copying. Similar effects have also been observed in Statistical Data Analysis 1. The most important aspect of the statistics quizzes for students is the doing of them, but mathematics and statistics staff often worry about the contribution to the overall result of components of assessment in which assistance is provided, and in which students can help each other. As described above, the format of the quizzes has encouraged constructive student interaction. Staff assistance focuses on helping students to find or work out the answers for themselves. Figure 2, illustrating the distribution of the totals for the five statistics quizzes in 2007, shows that although many students obtain, as expected, good scores, very few obtain top marks. The skewness to the left is due mostly to non-submissions. As in Example 1 (Statistical Data Analysis 1), there is also a strong positive relationship between the exam performance and the statistics quizzes, demonstrating their value in the student learning.

The group project data investigation is the same as in Statistical Data Analysis 1. This strategy was first developed for the engineering statistics units/modules and has been part of their programme since 1995. The engineering data investigation projects have been of a similar standard over the past decade. The projects are both significant learning and assessment tools. Because one-third of the project marks are for the ideas, plan, collection and quality of the data, and because assistance is available throughout the semester, failing project marks are associated with little effort. Sufficient effort and reasonable to good learning tend to produce project marks in the range of 12 to 17 (out of 20), depending on ability. Projects that achieve the highest marks demonstrate statistical thinking, skills and interpretation beyond what could be expected in an introductory unit. Figure 3 shows the project marks for 2007, demonstrating a shape similar to that of the quizzes in Figure 2, and that few projects achieve the highest marks.

In contrast to Figures 2 and 3, Figure 4, showing the distribution of marks on the numerical assignment, indicates problems. Far too many students chose not to do the assignment, and there are far too many high marks, indicating unhealthy collaboration. It became clear during assistance to students and during the marking that the assignment was too long, and its purpose within their learning was unclear to the students. Two additional external problems were that the assessment load in other units was heavy at the time, and, due to changes in the engineering course

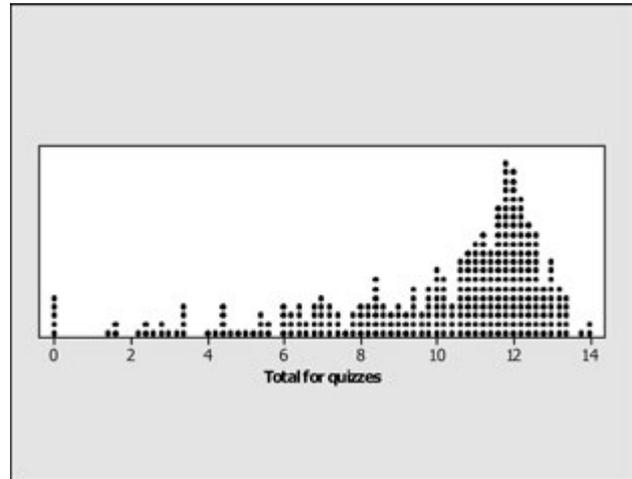


Figure 2: Plot of the total for the statistics quizzes in Example 4, a second year engineering unit, in 2007

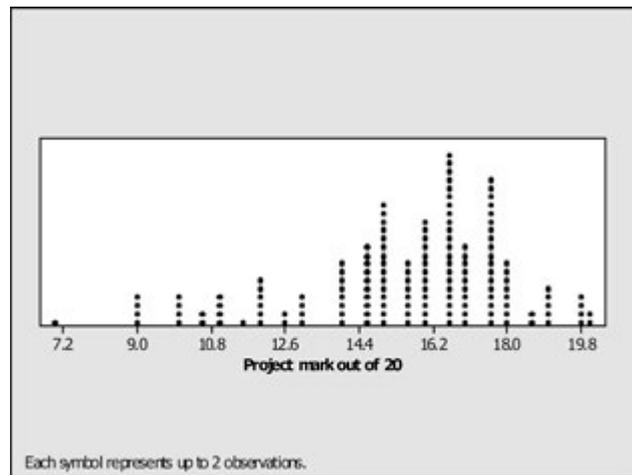


Figure 3: Plot of the project marks in Example 4, a second year engineering unit, in 2007

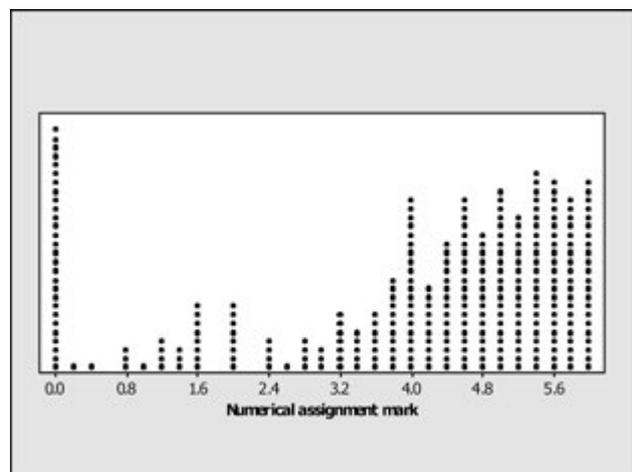


Figure 4: Plot of the numerical assignment marks in Example 4, a second year engineering unit, in 2007

structure, only some of the students had been introduced to Matlab where previously all had. The assignment could be done using Excel or Matlab, both of which are readily available to these students. However, because it took longer and was more tedious in Excel than in Matlab, the fairness of the numerical assignment was compromised by the changes in the engineering course structure. The combination of all these factors tended to lead to an inappropriate amount of collaboration with less learning.

The exam data and its relationships with components of the continuous assessment were also considered. The relationships between the marks on the data analysis section of the exam and the project and the relevant statistics quizzes were very similar to their equivalents in Statistical Data Analysis 1 as discussed above. The relationships between the numerical part of the exam and the numerical assignment, and the distribution questions on the exam and the relevant statistics quiz are shown in Figure 5. It is clear that the numerical assignment did not provide much help with student learning in this area, for reasons as discussed above. The second plot is indicative of 'burnout'. Although there is some positive relationship, the main feature of the plot is the increased variation of performance. Indeed, the students had heavy assessment tasks due in other units by week 10, 11 (due to a new "fad" rule in their faculty that 40% of assessment must be completed by week 11), so that even if they valiantly worked through the last quiz, many had difficulty in truly engaging with new work in the last few weeks of semester.

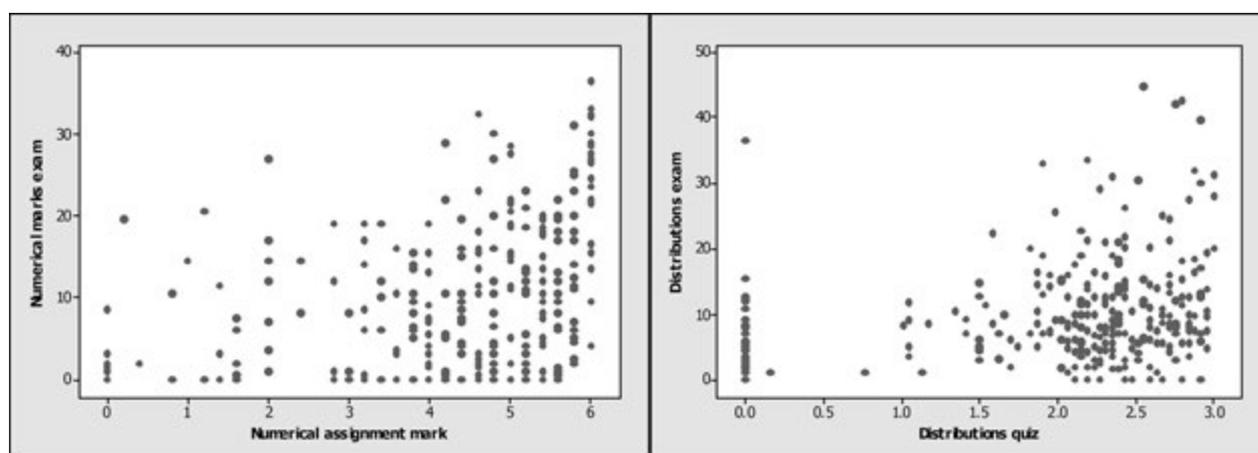


Figure 5: Plots of exam marks versus the corresponding assignment mark and quiz mark in the numerical section and the distribution section in Example 4, a second year engineering unit, in 2007

## Conclusion

In assessment for learning, each component of assessment has a role in an overall integrated, balanced, developmental and purposeful learning package. Assessment is aligned with learning objectives in an iterative and ongoing process that asks of each assessment component, what is of value that is being assessed, and of each objective, how is this objective learned and assessed. The learning and assessment package is structured for facilitation and management of student learning across the cohort diversity. Although the examples in this paper are for units in only one university, they have diversity over types of topics, objectives and cohorts. They illustrate how different components can be used and structured so that the combination aligns with an overall set of objectives, balancing formative and summative, group and individual tasks, assignments and tests, and workload. Selection of types and combinations of components of assessment depends on the nature of the unit. Identification of meaningful objectives specific to a unit can help staff in designing learning and assessment activities, and students in understanding their purpose. The importance of exploring, interrogating, analysing and interpreting data is emphasized and used to illustrate aspects of examples that were successful, and some assessment tasks with problems. All examples included group and individual work, with the group tasks being of the type that *need* a group, and being owned by the group. One example demonstrated the use of organised

collaborative work to facilitate learning in a problem-solving environment. Some interesting indicators emerged of characteristics of “homework” assignments in mathematics and statistics that appear to discourage copying on one hand, and not discourage it on the other. Over-long “homework” assignments whose learning purposes are not clear may increase tendencies to copy, in contrast with more pithy structures which are clearly designed for students to learn core operational knowledge and skills.

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# Using case studies in a mathematics tutor training programme

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## Abstract

We describe the development and use of materials for a mathematics tutor training programme at Dublin City University. This involved designing Case Studies for Maths Tutor Training, following the model pioneered in mathematics tutor education by Solomon Friedberg of Boston College. We describe how the Case Studies element was implemented in the tutor training programme, and discuss the feedback from trainee tutors on this and other aspects of the training programme both before and after their classroom practice.

## Introduction

The premise of the project described here is that a trained teacher is a more effective teacher [1]. The teachers in question here are mathematics tutors in Dublin City University (DCU).

Tutors in the School of Mathematical Sciences at DCU have undergone formal training since 1995. As Service Teaching Coordinator since 2002, the present author has had responsibility for the organisation of this workshop, which naturally led to a growing interest in how best to deal with the training of tutors. Thus, as with many colleagues in similar positions, I was greatly interested in an article in the August 2005 issue of the Notices of the American Mathematical Society entitled "Teaching mathematics graduate students how to teach" [2]. Here, Solomon Friedberg of Boston College, detailed his development of training materials that borrowed from business and legal education the idea of using the study and discussion of fictional but realistic scenarios as a means of accelerating experience and knowledge for tutors [3]. (In business and law, the cases are frequently taken from real life.) These training materials are gathered in [4]. This book contains 14 different case studies, along with a teaching guide for each which includes a synopsis of the case, a list of the educational issues encountered in the scenario, questions for the facilitator to put to the group of trainees to initiate and drive the discussion, and a question and/or activity to close the discussion. The book also contains general guidelines on the use of the cases in tutor training, as well as an account of how the cases were developed.

Two things were immediately clear: first, that this could be an excellent resource for tutor training. Second, there was a suspicion that there are sufficient differences between the US system and the Irish that Friedberg's cases might not be quite as relevant to the issues that arise here as one would like: Teaching Assistants in US colleges and universities tend to have wider ranging duties and more autonomy than tutors in Irish universities. Following up on the first of these (and reserving judgement on the second), Friedberg's ideas and materials were used in the DCU School of Mathematical Sciences Tutor Training Workshop in September 2005, on a pilot basis.

In the following account we describe this pilot project and the subsequent development of similar material in DCU that focussed more closely on the issues surrounding tutoring mathematics in an Irish university. We describe how this material was implemented in the tutor training programme of September 2006, and discuss feedback from tutors on the programme. We see the development and implementation of this material as fitting

in with other ongoing efforts to aid postgraduate students who are involved in front-line mathematics teaching – see for example [5].

The training material developed in the DCU Case Studies project is available online [6].

### **The Tutor Training Pilot Project**

Tutors in the DCU School of Mathematical Sciences are all post-graduate students in the School; in any given year, the new group of tutors consists of 6-10 students on a 1-year taught MSc programme, and 2-6 beginning research students. The School has 15 permanent staff members.

The aims of the tutor training programme are to prepare new tutors for their role by:

- giving a clear description of this role – and listening to their expectations of what the role involves;
- introducing the necessary communications and presentation skills;
- raising interest and enthusiasm for the task;
- giving an idea of what to expect in and around the classroom;
- stressing the importance of empathising with students.

The tutor training programme includes sessions on: the tutor's role in service maths teaching; communication skills and tips for tutoring; mathematical pedagogy; the Maths Learning Centre. In September 2005, a new element was introduced that used the case studies developed by Friedberg's team [4].

For any pre-service training programme, one can take the view that with the right foundation, effectiveness increases with experience. Case studies are used to accelerate the accumulation of knowledge and experience. The cases are used to generate a structured discussion of situations that may arise in and around tutorials. The analysis of different cases can help to identify important teaching and learning issues and to address how they can be tackled in the context of previous training and experience. They can help beginning tutors to anticipate problems that may arise. The focus is on general educational issues rather than the technicalities of being in a classroom.

Friedberg's team have developed 14 cases included in [4]. This book includes both the case studies themselves and a guide for facilitators. We identified and used three in the tutor training pilot project that seemed most relevant to our situation.

The group of ten trainee tutors were split into teams of 3, 3 and 4, with each team given responsibility for one case. The whole group read all three cases, and did some work on their own one in their smaller teams. We returned to the large group for discussion and analysis of each case. Each discussion was kicked-off by a synopsis, and answering of initial questions. The discussions were closed by an activity, piece of information, or summary question. (The facilitator's guide contains a list of questions that can be used to steer the discussion, as well as suggested wrap-up activities. For example, in a case involving plagiarism, the activity suggested is to give the University's policy on this topic and to discuss its implementation.)

In this pilot project, the trainee tutors' opinions on the general programme and the case studies in particular were sought:

*"very enjoyable...very interesting debate"*

*"most relevant [part of the day]"*

*"I was only interested in the case study allocated to my group"*

*"we could have covered a wider range of situations if they had been studied in less detail"*

*“did not find the particular case studies relevant”*

*“when we were discussing the tutorials amongst ourselves was the most useful”*

This selection of comments is representative of those made by the trainee tutors as a whole, which were generally though not universally positive. One important feature that was not referred to, but that the present author observed, was this: for each case, there was a very lively debate, with every trainee tutor present reflecting on the situation under discussion and giving their opinions on it. Thus it was clear that the use of case studies provided an effective means for encouraging trainees to reflect on their role: an important aspect of pre-service training that cuts across the aims of the programme listed above.

It was also clear that in practice, the differences between the Irish and US situations meant that the material of [4] was not ideally suited to the needs of our tutor training programme. It was decided that similar training material would be developed locally.

### **The DCU Case Studies project**

Following discussions with colleagues, it was decided that a Case Studies project could be usefully expanded to cover tutor training for beginning tutors and laboratory demonstrators – all referred to as ‘tutors’ henceforth – across four Schools of DCU’s Faculty of Science and Health: Mathematical Sciences, Chemical Sciences, Physical Sciences and Biotechnology. In March 2006, funding from DCU’s Learning Innovation Unit was obtained to develop ‘Case Studies for Science and Mathematics Tutor Training’. The project team consisted of the author, Odilla Finlayson (Chemical Sciences), Eilish McLoughlin (Physical Sciences), Michael Parkinson (Biotechnology) and 11 experienced tutors from across the four Schools. The method of development of the training material followed that of Friedberg [4].

A first Project Team Workshop was held in May 2006. In this, the Project Team identified – through a pyramid discussion – the 20 most important issues facing tutors in DCU. The list arrived at on that day was:

- Expectations (students’ of the tutor and vice versa).
- Different tutoring styles; running a tutorial.
- Knowing the students’ background.
- Boundaries.
- Empathising with the student.
- Conflicting instructions.
- Relationship with the class: authority, respect and trust.
- Handling difficult questions.
- Presenting concepts.
- Cheating and plagiarism.
- Dealing with mixed abilities.
- Apathy versus enthusiasm.
- Questioning and listening skills.
- Giving students feedback (in class and on assessed work).
- Health and safety.
- Exam versus education.

- Dealing with difficult students.
- Grading assessed work.
- Self-evaluation of the tutor's role.
- Time management.

Next, the Project Team split into four groups of 3-4, with each group being given responsibility for a subset of the 20 issues listed. Their task was to write the synopses of three different scenarios (case studies) in which each of their issues was present at least once. These synopses were then discussed by the whole Project Team, and subsequently revised. The facilitator briefly went through some material on grammar and creative writing, which was distributed to the groups. Each group then had five weeks to write a full draft of their case studies, running to around 1000 words, and to develop the accompanying teaching guide (working to a given template that replicated Friedberg's model).

In the second Project Team Workshop in July 2006, each case study was given a test-run by a group not involved in its preparation. The key quality being sought was that each case study generated a discussion about the issues that it sought to highlight. The case studies were then revised. The final versions were completed and collected by the beginning of August 2006. See [6]. To give a flavour of what these case studies look like, we reproduce the synopsis of one of them:

### **Case Study: Patricia's Practical Problem**

*Patricia is a first-year post-grad student. Her teaching duties include the supervision of fourth year labs of the course she just graduated from. This class contains many of her friends, two of whom, Claire and Elaine, consistently hand up suspiciously similar lab reports. She knows that she must deal with this issue but she is afraid of jeopardising their friendship.*

The accompanying teaching guide includes the following questions, aimed at driving the discussion of the case.

1. *Should Claire and Elaine be punished? Should they just be warned personally? Or should the whole class be warned? Who should Patricia report this matter to?*
2. *Was Patricia right to bring the matter up informally at first?*
3. *Should Patricia have been assigned to this class? Should tutors be asked to grade the work of students who are their friends?*
4. *What guidelines should tutors be given for grading work?*
5. *What would be a fair and reasonable policy on plagiarism?*

The material developed was used in the DCU Faculty of Science and Health's tutor training programme in September 2006. This two-day workshop included two sessions in which the Case Studies material was used. These were discussed in groups of 10-12.

### **Feedback**

In Table 1, we summarise responses on a Likert scale from the mathematics tutors who undertook the tutor training programme in September 2006. This survey was taken before the trainees began work as tutors. The responses refer to the programme as a whole, and so it is difficult to draw conclusions about the Case Studies aspect in particular. That said, it was specifically this element of the training programme with which we sought to increase the trainees' levels of awareness of their role and its importance, awareness of the importance of

empathising with students and their confidence levels, and so the positive responses on items A, B, D and E can be interpreted as providing evidence of the success of the Case Studies element of the programme.

	<b>The training workshop has increased...</b>	Agree Strongly (%)	Agree	Neutral	Disagree	Disagree Strongly
A	...my awareness of what to expect in labs/tutorials	14	86	0	0	0
B	...my awareness of the importance of my role as a tutor.	57	43	0	0	0
C	...my interest in and enthusiasm for tutoring	43	57	0	0	0
D	...my awareness of the importance of empathising with students.	29	57	14	0	0
	<b>Overall, I think the training workshop...</b>	Agree Strongly (%)	Agree	Neutral	Disagree	Disagree Strongly
E	...will help to make me a more effective tutor.	43	57	0	0	0
F	...was relevant to the job I will be doing as a tutor.	43	43	14	0	0
G	...helped me to learn some valuable skills.	43	57	0	0	0

Table 1: Feedback post-workshop, pre-practice (2006).

Open-ended responses also showed evidence of the utility of the Case Studies element:

*"helps get to know one another...hearing [the other new tutors'] ideas and experiences is also very helpful"*

*"case studies...often a bit exaggerated, but helpful nonetheless"*

*"most helpful part...was the case studies...they address problems which would have thrown me before, but I now think I'd be better able to handle them"*

Feedback from the mathematics tutors was also sought after 10 weeks of their first semester of tutorial work. A round-table discussion was held, at which the following points emerged:

- The workshop as a whole provided good preparation for working as a tutor – helped to allay nerves;
- Case studies were realistic, and helped with foreseeing and dealing with different problematic situations;
- Questioning/listening skills, tips for tutoring and presentation skills were of most practical benefit.

Again, reaction to the Case Studies was broadly positive. As with the pilot project, one important aspect of the training programme was not reported by the tutors, but was clear to (although not anticipated by) the present author: the tutors had created a 'community of practice' among themselves. This community appears to have emerged from the discussions about tutoring that began in the training programme, and that continued throughout the semester.

## Conclusions

We have argued that the tutors' opinions reported above speak positively of the benefits of the Case Studies element of the tutor training programme we have described. These benefits could perhaps be summarised in a twofold manner: on the one hand, the tutors reported that the programme helped to reduce their anxieties about tutoring and on the other, the programme helped the tutors, through their own creation of a 'community of practice' and in other ways, to become reflective practitioners [7].

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# Mathematics support as a practical discipline

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## Abstract

This paper explores the status of Mathematics Support as a practical discipline. The emergence of Mathematics Support as a practical discipline in UK Further and Higher Education is described by reference to its historical origins, through surveys of the extent of provision within UK Higher Education Institutions and through centralised developments in the UK. The Mathematics Support service provided by the *sigma* Centre for Excellence in Teaching and Learning is described. Theories of academic disciplines by Becher and Craig are introduced and an analysis of the discipline of Mathematics Support is provided with reference to these two theories. Finally, conclusions are drawn into the status of Mathematics Support as a practical discipline.

## The Emergence of Mathematics Support as a Practical Discipline

### Definition of and Rationale for Mathematics Support

*Mathematics Support* is a term used mainly in UK Further Education (FE) and Higher Education (HE) to describe the provision of supplementary forms of teaching and resources for mathematics learning across institutions in addition to the main teaching provision. According to [1] and to [2] (p. 3) common forms of Mathematics Support are:

- *Bridging courses;*
- *Computer-aided learning;*
- *Diagnostic testing* (normally at university entry, and normally followed by other forms of follow-up support);
- *Drop-in centres* (alternatively known *workshops*);
- *Numeracy classes;*
- *Paper-based open learning materials;*
- *Peer study support;*
- *Tutoring;*
- *Videos;*
- *Websites* (which may also include some of the above types of resources)

(Another important term here is *Mathematics Support Centres* or *Mathematics Learning Centres*: this refers to a drop-in centre room from which other forms of support may also be provided.)

The main reason for the emergence of Mathematics Support provisions is *the mismatch between students' mathematical confidence, knowledge and skills at university entry and those required in order to commence their degree courses* [3] (p. 5). A secondary reason for its emergence is the *increasing breadth of variation of*

mathematical and statistical competences of students entering the same university courses. As lecturers tend to target the average (or slightly below average) student ability, this means that not only 'at risk' students with poor entry skills require supplementary support, but support also needs to be provided for brighter students who are not sufficiently challenged [4].

### History of the UK Mathematics Support movement

Although some mathematics drop-in centres started in UK HE Institutions (HEIs) around 1990, the origins of the UK Mathematics Support movement can be traced back to the First National Conference on Mathematics Support hosted by the University of Luton in 1993. This conference provided presentations in the area of Mathematics Support, feedback on the first national survey of mathematics support provision [5], and a model of how support might be developed based on the academic support centre at Minnesota General College (now called Minnesota College of Education and Human Development) [6]. It also led to the establishment of the Mathematics Support Association which ran from 1993 to 1999 and produced eight newsletters.

Some Mathematics Support Centres, such as those provided at Coventry and Loughborough Universities, were originally set up to support engineering mathematics teaching. However, many now support mathematics teaching university-wide (again, see [3], p.5 for a rationale).

This historical association with engineering mathematics may also explain the greater emphasis on mathematics and the lesser emphasis on statistics within the discipline. Whilst statistics support has generally been provided within mathematics support centres, statistics was generally not perceived as a subject requiring a specialist approach until the emergence of Statistics Advisory Services as part of the **sigma** Centre for Excellence (see below).

### Extent of provision of Mathematics Support in UK FE and HE

A summary of the published surveys into the extent of the provision of Mathematics Support in UK FE and HE is shown in Table 1. Of these, the surveys of Beveridge in 1996 and Perkin and Croft in 2004 were the most thorough in terms of using several forms of inquiry and attempting to approach several people at each institution. From these two surveys we conclude that at least half of all UK HEIs have a Mathematics Support drop-in workshop or drop-in centre.

Investigator(s)	Date	Reference	Type	Scope	Proportion offering a drop-in workshop
Beveridge & Bhanot	1993	[5]	Questionnaire	800 FEIs & HEIs contacted with 100 FEIs and 42 HEIs replying	76%
Beveridge	1996	[1]	Telephone & postal surveys	Replies from 150 HEIs & 50 FEIs	56%
Beveridge	1999	[7]	Not stated	Not stated	77%
Lawson, Halpin & Croft	2001	[8]	On-line questionnaire	Replies from 95 HEIs	48%*
Perkin & Croft	2004	[9]	Email & telephone survey	Replies from 101 HEIs	65%*

\* Refers to Mathematics Support centres rather than drop-in workshops

Table 1: Published surveys into UK FE and HE Mathematics Support provision

## National Mathematics Support initiatives

After Lawson, Halpin & Croft's survey in 2001 the Learning and Teaching Support Network (LTSN, now superseded by the Higher Education Academy subject centre) in Mathematics, Statistics and Operations Research (MSOR) funded an in-depth case-study research into Mathematics Support in general [2] and diagnostic testing in particular [10].

After this, LTSN funded the development of *mathcentre* – a website providing a variety of types of integrated resources for students in the transition from school to FE or HE, some of which are contextualised [11]. About the same time, the Higher Education Funding Council of England (HEFCE) Fund for the Development of Teaching and Learning, the Gatsby Foundation and the EBS Trust jointly funded *mathtutor*. Originally distributed on DVD, this is now a website based around streamed videos which teach generic mathematical subjects. Both these sites – *mathcentre* and *mathtutor* – have become extremely popular, both nationally and internationally, with *mathcentre* having up to 330,000 hits per month [12].

## The sigma Centre for Excellence in Teaching and Learning

Coventry and Loughborough Universities are acknowledged leaders in the field of university-wide (i.e. specialist and non-specialist) Mathematics and Statistics Support. Their substantial experience has been harnessed in a successful bid to HEFCE to establish a Centre for Excellence in Teaching and Learning (CETL) in *the University-wide Provision of Mathematics and Statistics Support*, called **sigma** [13].

**sigma** began operating in 2005 and has funding for five years. Its main aim is to develop mathematics and statistics support and to incorporate innovative approaches to teaching and learning that address the widely differing curricula and individual needs of different students. Its activities are being underpinned by a systematic programme of educational research.

**sigma** is making a substantial investment to enhance existing provision in the two Universities and to address proactively the needs of those who can benefit from the support available. Its activities include:

- extended and enhanced drop-in centres at the two universities;
- a Statistics Advisory Service offering individual statistical support, e.g. for final year project students and postgraduate students [14];
- a proactive teaching interventions programme which aims to target 'at risk' students within large cohorts [15];
- investigation and development of innovative uses of technology in Mathematics Support, such as the use of classroom communication systems and educational computer games.

## Theories of academic disciplines

In this section we shall consider two theories of academic disciplines.

Firstly, one of the currently most influential theories of academic disciplines is Becher's *tribes and territories* [16] in which he describes academic disciplines in terms of two perspectives:

- *the social aspects of knowledge communities (the academic culture of the discipline); and*
- *the epistemological properties of knowledge forms (the nature of knowledge within the discipline).*

His theory was established through interviewing academics from 12 different disciplines in UK HE, including mathematics.

Becher concludes that these two perspectives “are so inextricably connected that it is unproductive to forge any sharp division between them” [16] (page 20). He also emphasises the importance of “international currency” (i.e. the academic exchanges between different countries and cultures practising the same discipline) for the recognition of disciplines.

Secondly, Craig agrees with Becher but asserts that disciplines should also be viewed from a third *sociocultural* perspective which he defines as the degree to which “ordinary concepts and practices” are “ingrained in the cultural belief systems and habits of the society at large” [17]. He also distinguishes *practical disciplines* from *theoretical disciplines* by defining the former as those that are “reflexively engaged in the cultivation of the very social practices that constitute the discipline’s subject matter”. He then asserts that practical disciplines grow to prominence because they “credibly purport to be useful in addressing a range of practical concerns already acknowledged as such in society”. Craig then argues that *Communication Studies* is such a practical academic discipline.

In the next section, these two theories are applied to Mathematics Support in the light of the previous section describing its history, development and current status. However, both these theories are somewhat *static* rather than *dynamic* as neither describes adequately how academic disciplines form and grow in strength and authority.

### **Application of academic discipline theories to Mathematics Support**

From these theories we make the following observations and assertions about the legitimacy of viewing Mathematics Support, especially within UK HE, as a discipline in its own right:

1. Using Craig’s definition, Mathematics Support is a *practical discipline* as it has arisen out of the practice of supporting students with mathematical difficulties in FE and HE.
2. In terms of *sociocultural relevance*, there is a strong social justification for the existence of Mathematics Support from sources including UK Government reports into the mathematics “problem” in HE, such as [18], and objective evidence of falling standards in mathematics, such as [19]. The need for extracurricular academic support in mathematics, especially when starting at university, remains stronger than ever. The existence of Mathematics Support as a separate subject to mathematics and mathematics teaching can be compared to the existence of *Educational Development* as a separate academic discipline – it is the need to improve current educational practices that justifies its existence.
3. However, even within the community of practice of Mathematics Support there have been practitioners who disagree with it being viewed as a discipline. For example, Cock considers the whole issue of Mathematics Support to be misconceived as it shifts too much responsibility away from mathematics lecturers [20].
4. Mathematics Support derives some legitimacy in UK HE from its *institutional* existence (as shown by the above surveys) and *national resources* (as described above).
5. There is a growing *literature* on good practice in Mathematics Support provision, for example [3]. However, this literature has not yet been accompanied by a widespread professional training in Mathematics Support tutoring in UK HE (although the same is also true for subject specific training in mathematics teaching in UK HE, which is also in its infancy – see [21]). We do, however, note that the Minnesota College of Education and Human Development has produced a handbook on “peer tutoring in the mathematics workshop” [22] and Loughborough University has recently run a Postgraduate Certificate in Mathematics Support and Dyscalculia in FE and HE [23].
6. **sigma** has a growing Mathematics Support *research community* with four funded PhD students currently in post [24]. This reflects favourably against the UK HE mathematics educational research community which is very small in relation to the number of practitioners of HE mathematics teaching.

7. Since the demise of the Mathematics Support Association, the Mathematics Support community in the UK has not had such a strong *sense of identity and community of practice*. However, the development of *mathcentre* and *mathtutor* practically demonstrated community activity. The award of CETL status to Coventry and Loughborough Universities has raised the profile of the Mathematics Support discipline with the hope of building a stronger community of practice through the lifetime of the CETL (i.e. by 2010).
8. On the issue of *international currency*, a Network of Mathematics Learning Support and Research Centres was established in 2002 [25]. Apart from the historical links with Minnesota General College already mentioned, **sigma** also leads a European consortium that is developing websites similar to *mathcentre* for other European countries [26].

## Conclusions

This paper has given an account of the history and development of Mathematics Support in the UK. With reference to Becher and Craig's theories of academic disciplines, Mathematics Support may be viewed as a practical discipline which forms an important subset of UK FE and HE institutional-wide mathematics teaching (in a broad sense), although it is still at a formative stage of development. The provision of more training in Mathematics Support tutoring, stronger international links and peer evaluation of both Mathematics Support tutoring and resources would increase the *authoritative voice* of the discipline.

It is anticipated that Mathematics Support will continue to exist and grow in maturity because the rationale for its existence is likely to remain and strengthen in the future. The award of CETL status to **sigma** and other national collaborations in Mathematics Support has also strengthened its status.

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# Mathematics support for students on vocational courses

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## Abstract

The mathematics support service at London South Bank University was set up in 1998 and with this came the first formal, term-time, support for mathematics and statistics across the University. Prior to this, support of this nature was contained, if at all, within each separate School/Faculty at the University.

In contrast, at Birmingham City University mathematics support has been developed within the Faculty of Health to specifically cater to the needs of healthcare students. This support includes a full-time post of a dedicated numeracy support tutor within the Faculty's Personal Development Centre.

A number of factors need to be taken into account when running a support service. For example, important considerations are the finance and staffing of the service. The type of mathematics required for nurses, trainee teachers and social workers is very different to that required for those studying engineering and science based subjects. Running a good support service requires well-qualified and experienced staff in this area.

This paper briefly describes the two mathematics support services and compares and contrasts how each service is financed and staffed. The benefits, constraints and problems of providing both types of support service for students, particularly those on vocational courses, are discussed.

## London South Bank University

During 1998 London South Bank University (LSBU) made a strategic decision to fund a central academic support service for all students entering its courses. The support service was designed with the University's commitment to 'widening provision' in mind and had the aim of enabling progression for local people. Southwark, Lewisham and Lambeth are working class boroughs which have some of the lowest progression rates to Higher Education (HE) in the country. Few people from these boroughs have had any experience of HE and many adults lack formal qualifications. The support service provided is in addition to the regular lectures and tutorials that students receive as part of their course at the University. Some of this central academic support had been provided, for several years, in a rather ad hoc manner. For example mathematics and statistics support used to be provided by the School of Computing and Mathematics. Since the mathematics degree was being discontinued, there was concern that experienced teaching staff may be lost, leaving a small pool of staff that would have insufficient time and resources to provide this type of support. Mathematics and statistics support was considered essential to enable students under the 'widening participation' umbrella to be successful at LSBU, and so a central support service for mathematics and statistics was thus set up as part of the Skills for Learning Team [1] within the LSBU's Centre for Learning Support and Development (CLSD).

The CLSD provides the following services – Library Services, IT support, Skills for Learning, Student Advice and Guidance, Disability and Dyslexia Support.

The Skills for Learning service within CLSD provides both Mathematics and Statistics support and English Language support for all students at LSBU. The Maths Support Programme (MSP) has expanded since 1998 and now runs workshops, drop-in sessions, nursing numeracy sessions, PGCE sessions for trainee teachers, maths for pre-entry social workers and one-to-one appointments. Support is provided on every weekday and a timetable of support is published. MSP also undertakes bespoke sessions on request from faculty lecturers. During the 2005-06 academic year MSP provided support for approximately 1400 students. Note that there is some double-counting of students who attended more than one set of classes. (See [2] for the published programme of support sessions.) MSP also had 500 students who attended bespoke sessions provided. By way of contrast, back in the 1998-99 academic year there was only 9 hours per week of mathematics support provided for a total of 238 students.

MSP also provides a 6-week summer mathematics programme for 350 potential and existing students. Students can attend this course every day for six weeks to bring their mathematical and statistical skills up to the level required to enter or continue at LSBU.

The Skills for Learning Team in total consists of three full-time academic staff, several part-time academic staff, a full-time administrator and approximately 50 term-time hourly paid lecturers who work throughout the year. Strengths of the service are (a) a student-centred teaching approach, (b) a good informal staff development programme for teaching staff, (c) a Blackboard virtual learning environment site that has teaching resources which can be used by teaching staff and students alike and (d) a close working relationship with the academic staff within the four Faculties at LSBU. Student usage levels have increased year on year.

The MSP sees students from across all the Faculties at LSBU, which provides interesting mathematical/statistical examples, e.g. from nursing, teaching, social work, engineering and postgraduate studies. The types of questions/help requested are:

- basic numeracy;
- drug calculations;
- statistics;
- calculus;
- on-line maths tests for accountants.

MSP have in-house resources such as worksheets and tests and also use the mathcentre website [3] and the *mathtutor* DVD.

LSBU is a teaching-led institution and hence undertaking research on resources to aid teaching is paramount; the academic staff's personal research has led to the MSP having a bank of relevant and useful material that the staff can share and use. Problems in setting up and running the MSP which have had to be overcome are:

- The need for a large pool of staff able to teach at the level require (i.e. from pre-entry right up to postgraduate);
- The need for a full time administrator as the service expanded;
- Dealing with staff holiday issues due too running all year round;
- The inevitable need for appropriate secure funding;
- The need for relevant teaching material and resources;
- The need to provide adequate evidence and evaluation of success of the service to justify its continued support.

The MSP has its own dedicated teaching rooms that are equipped with up to date Smart Media Board technology. Quotes from students who have used our service are extremely positive such as:

*'I do appreciate what the tutors did for me.'*

*'The best aspect of the support class was the amount of knowledge I have attained in such a short period of time.'*

It is clear from these and many similar comments that the students value the service and in particular the personal contact with a member of staff who has the required mathematical and statistical expertise. This is an expensive resource to fund but if the MSP is to help LSBU's students to succeed then it is an essential service.

The MSP staff keep electronic registers of the students who use the service. This can be used for statistical justification for maintaining funding to run the service, and so that it can be seen which sessions should be repeated or retained for subsequent academic years because they are so popular or removed if not required. In the one-to-one sessions staff record the topic for which help is requested; if this topic request is repeated many times then for the next academic year it is put on as a scheduled session. Hence the Mathematics Support Programme is dynamic rather than static in the provision offered.

### **Birmingham City University**

Mathematics support arose from within the BCU Faculty of Health in response to the specific needs of the students. It is delivered from within the Personal Development Centre (PDC) which is a Faculty of Health Department that caters for much of the academic support within the Faculty as well as monitoring and administering Accreditation for Prior Experiential Learning (APEL).

In September 2004, a full time member of academic teaching staff was appointed to the PDC with a specific remit to support students with their numeracy. The post holder's background is in dyslexia support and much of his approach is influenced by this; he recognizes that mathematics is an area that causes many people anxiety and endeavours to improve confidence and competence in using mathematical skills by being non-judgmental and encouraging.

The support is available to all students within the Faculty; however it is predominantly accessed by DipHE and BSc (Hons.) Nursing programmes, BSc (Hons.) Midwifery and Graduate Diplomats. Although the entrance requirements vary for these groups of students, the nature of their difficulties with mathematics is rather homogeneous. Many present with low confidence and a feeling of inadequacy, often fuelled by poor experiences of mathematics education and/or a long time lapse since the skills were practised.

The focus on numeracy has risen within the Faculty in response to concerns of the partner NHS Trusts as to the level of numeracy competence of newly qualified nursing staff. In 2006 a Working Party was convened to develop a Faculty Numeracy Strategy which would address the concerns of both academic and clinical staff. Three major initiatives comprise the strategy - more numeracy input in programmes; more numeracy support for students, and staged summative numeracy assessments.

The staged summative assessments have been implemented to help ensure that students are updating and developing their numeracy skills appropriately in order to fulfill fitness for practice criteria. As competent numeracy skills are necessary employment prerequisites, it follows that these skills need to be assessed on a regular basis as it cannot be assumed that all students are competent as a matter of course. As anticipated, the implementation of the Numeracy Strategy has led to a significant increase in the uptake of support as students are compelled to deal with their difficulties at a more opportune time.

The nature of the Faculty's mathematics support has evolved markedly in three years. In its current form it consists of the following strands:

- *Self-assessment:* Students are encouraged to identify their learning needs and seek help accordingly. In some cases they are given self-assessment numeracy quizzes to inform their decision as to whether support is needed.
- *Evening workshops:* A series of evening numeracy workshops are repeated throughout the year. They cover the basics of number computation.
- *Consolidation and Preparation workshops:* At certain periods in the year BSc Hons. and DipHE Nursing students have space in their timetables to attend academic development workshops designed to enhance areas of academic skill that they have identified as weaknesses. These have taken on two forms where numeracy is concerned. For students who have reached the end of their first year on programme, they can elect to attend numeracy revision workshops. For those students who have reached the end of their second year on programme they can elect to attend Drug Calculation revision workshops.
- *Module specific exam preparation workshops:* These are designed to give students a feel for the numeracy exam conditions.
- *Lunchtime drop-in:* Traditionally within the PDC students have been able to access drop-in support for such areas as academic writing, critical analysis planning essays, etc. Students can now also drop-in for support in numeracy.
- *Bespoke provision:* Students can attend one to one or small group tutorials by prior appointment.
- *Online support:* BCU uses Moodle as its virtual learning environment. The numeracy support tutor is the facilitator of a Moodle site entitled 'Drug Calculation Numeracy Skills'. This is a repository of basic numeracy information and practice exercises for students to assess their numeracy competence. In November 2006 the numeracy support tutor was made a Faculty Teaching Fellow with a project to further develop a Moodle and paper-based numeracy resource.

Four members of staff are now involved in delivering various aspects of the above support.

## **Benefits, Constraints and Problems**

The support services offered by both LSBU and BCU provide valuable help for students. The problems associated with providing any support service are numerous, whether faculty or centrally based. The issue of having experienced staff on hand able to answer questions is essential. This is easier to handle if based in a faculty with a person appointed with a specific remit of supporting students in that faculty (as in the case of BCU, for Health Faculty students). Staff who can help students who come from across the whole university, as in LSBU, are even more difficult to find than lecturers in a defined subject area.

The question of providing ongoing support rather than just one-off support must be addressed, and is an essential requirement for the students. In addition, the lecturers/tutors have to be fully conversant with teaching pedagogy as well as learning and assessment techniques.

Funding is vital for any support service to survive. At BCU it is funded from the Faculty of Health and at LSBU the funding comes from top-slicing all the faculties i.e. part of their budget is automatically allocated to the support service budget. This top-slicing means that the LSBU service is able to undertake long term planning in the secure knowledge that money is being provided.

Students at LSBU like coming to the central support service as they (a) mix with other students across the University and learn from each other, (b) are confident that their faculty lecturers do not know they have extra help, (c) can ask questions about the other services provided at the same time and (d) can have longer time with experienced academic staff on their problems.

The benefits of the BCU model of numeracy support are in its specialization and flexibility. In addition, being based within the Faculty, PDC staff can influence its curricular and assessment strategies. The specific nature of this support means that tutors can focus on a specialized area of numeracy. The support appeals to students who wish to attend in small groups as well as those seeking one to one sessions. There is online provision for those students who may want to explore their numeracy competence independently, but it can also enhance face to face sessions.

Many of the students, from both Universities, have very little confidence in their mathematical abilities and feel inadequate in attempting questions involving calculations. The prior experience these students have had in mathematics education is often poor. Also, many of them are returning to education after some considerable time. Both Centres offer support that is non-judgmental and recognizes that mathematics is an area that causes many students grief.

The following are future considerations for the continued implementation of Numeracy Support at LSBU and BCU:

- The mathematical competence of students as they enter programmes/courses at university;
- Staffing levels and hence funding;
- Relevant experience of the staff providing support (i.e. requiring expertise in both mathematics and teaching pedagogy);
- Well-maintained online support such as Blackboard or Moodle as a virtual learning environment;
- Appropriate resources;
- Because of the vocational courses that both Universities offer, the academic input must maintain its relevance and plenty of opportunities need to be made available for students to consolidate learning with practical hands-on experience.

Finally, it is encouraging to report that students at both Universities have been well pleased with the support service they have been provided with and have requested more of the same.

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3. <http://www.mathcentre.ac.uk/>

## Promoting engagement with mathematics support – understanding why students don't come

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### Abstract

The Mathematics Learning Support Centre (MLSC) at Loughborough University was established in 1996 to provide assistance to engineering undergraduates with mathematics learning needs. However, recent MLSC usage data indicates that many students who need mathematics support are not using the Centre. This paper reports some findings from focus group and interview research, which seeks to identify the reasons why these students are not availing themselves of the support offered by the MLSC. From the first phase of this research, the paper discusses the reasons expressed by seven first and second year science and engineering students, who had failed a first year mathematics module. One of the main reasons discovered is a lack of awareness of the need for support. Subsequently, the paper uses data from the second phase of this research to discuss the reasons expressed by 84 students from across the campus. These reasons include a lack of awareness of the location of the MLSC and its facilities and an uncertainty as to whether it is relevant to their courses. The paper will use these findings to suggest possible action to improve the uptake of support.

### Introduction

It is widely accepted that there has been a decline in the mathematical preparedness of students on entry to universities in the UK and that many students embarking on a degree course lack some basic mathematical skills [1, 2]. A popular strategy of supporting students is in the form of a mathematics support centre, whereby learning support is offered to students, which is additional to that provided by their normal teaching. In 2004, Perkin and Croft [3] found that 66 out of 106 UK universities surveyed provided mathematics support.

At Loughborough University mathematics support is offered by the Mathematics Learning Support Centre (MLSC). It provides a wide range of support mechanisms including one-to-one support on a drop-in basis, paper-based handouts and computer-based material. Due to the MLSC's success in supporting students and similar work at Coventry University, both Loughborough University and Coventry University were jointly awarded Centre for Excellence in Teaching and Learning (CETL) status in 2005. A new centre, **sigma**, has been established between the two Universities and the funding that the CETL award brings is currently being used to expand and enhance the provision of mathematics and statistics support.

The MLSC at Loughborough University is highly valued by staff and students and recognised as an integral part of the University [4]. The success of the MLSC is evident through its popularity amongst students, with a recorded 3926 visits in 2005/6 [5]. However, analysis of recent MLSC usage data has revealed that a large proportion of Science and Engineering students who need mathematics support are not using the Centre. In particular, data from 2005/6 reveals that of 626 Engineering and Physics students taking a first year mathematics module, 96 failed at the first attempt. Of those who had failed, it was found that over 90% (or 87 students) had never, or very

rarely, accessed the extensive support available via the MLSC. Support provided by the MLSC requires students to be *proactive* and take the initiative in accessing the support available. Consequently, if students are unaware of their weaknesses or lack motivation to seek support, then the support will remain unused. Therefore, it is essential that the reasons behind the lack of uptake of support are identified so that appropriate action can be taken to improve it.

This paper describes a study conducted in the academic year 2006/7 which sought to identify reasons why failing students do not use the MLSC. It will give details of the study itself including the participants and the methodology used. Data from the focus group and interviews will then be analysed and the results of these will be discussed in detail. The paper will then use these findings to suggest possible action to improve the lack of uptake of support.

## The Study

### Methodology – Phase 1

Current first year and second year students who had failed a first year mathematics module at the first attempt (in 2005/6 and 2006/7) and who had never or rarely used the MLSC were contacted via e-mail (on three separate occasions) and were invited to take part in a focus group session. However, out of 179 students, only seven students responded. This apparent lack of willingness to share the student perspective may indicate that the students who had failed the mathematics module, in particular students with a 'hard' fail (i.e. achieved less than 30%), were not comfortable in discussing their thoughts about mathematics or their ability in this subject.

With such a small number of students, and given time restrictions and other practicalities, only one focus group session was conducted (with 3 students). Two of the remaining four students were interviewed in a group setting and the remaining two students were interviewed individually. A profile of the seven students can be seen in Table 1. In the focus group session and the interviews a number of pre-determined questions were put to the participants. All sessions were led by one of the authors of this paper (Symonds) and the discussions were recorded using a digital voice recorder.

Student	Year	Course	Maths module mark %*	Number of visits to the MLSC	Session
A	2	Physics	33%	3	Focus Group
B	2	Electrical and Electronic Engineering	34%	2	Focus Group
C	2	Electrical and Electronic Engineering	33%	0	Focus Group
D	1	Science and Engineering Foundation	20%	0	Group Interview
E	1	Science and Engineering Foundation	32%	1	Group Interview
F	1	Physics	33%	0	Individual Interview
G	1	Physics	35%	0	Individual Interview

\* Marks below 40% are deemed as a fail and marks below 30% are deemed as a 'hard' fail.

Table 1: Phase 1 Student Profiles

## Methodology – Phase 2

Since the email invitation method of recruiting participants for focus group sessions and/or interviews had very limited success, it was decided that a different method would be tried to obtain more data. This involved approaching various students around the University campus and questioning the students using a survey interview technique. A condensed version of the interview questions from Phase 1 was used. The same questions regarding the MLSC and usage of the support were asked in both phases and these are analysed below. The responses were recorded in writing by the researcher, since this approach allowed the students to give short precise answers that could be quickly recorded. This also allowed the researcher to probe any interesting comments made by the student.

This method was conducted on three separate occasions. A sample of 30 and 33 students respectively were approached in the library on two occasions, during separate revision weeks of the academic term. An additional sample of 22 students was approached in the Student’s Union and Canteen, also during a revision week. All students had studied a mathematics or statistics module.

Out of the additional 85 students questioned, 12 were later identified as having failed a mathematics module *and* had rarely or never used the MLSC. Since this study wishes to identify the reasons why failing students fail to engage with mathematics support, comments made by these students will be analysed in detail. A profile of these twelve students can be seen in Table 2.

Student	Year	Course	Maths module mark %*	Number of visits to the MLSC	Sample
H	1	Financial Mathematics	26%	0	Library 1
I	P1**	Banking and Financial Markets	29%	0	Library 1
J	1	Geography	33%	0	Library 1
K	3	Sports Science and Physics	23%	1	Library 2
L	2	Mathematics	34%	3	Library 2
M	2	Automotive Engineering	33%	0	Library 2
N	2	Automotive Engineering	28%	0	Library 2
O	2	Aeronautical Engineering	31%	0	Library 2
P	2	Electrical and Electronic Engineering	37%	0	Library 2
Q	2	Civil and Building Engineering Foundation	26%	2	Library 2
R	2	Product Design	36%	0	Library 2
S	1	Systems Engineering	19%	1	Union

\* Marks below 40% are deemed as a fail and marks below 30% are deemed as a ‘hard’ fail.

\*\* P1 = Postgraduate first year

Table 2: Phase 2 Student Profiles

## Sampling

The sample used in this study cannot be considered as being a random sample, however it may be regarded as a ‘typical’ sample, in that the students who responded may typify some of the attitudes of general non-users of the Centre. It should be noted that measures were taken to reduce bias as much possible. Students were not solely recruited on a volunteer basis; “on the spot” interviews secured responses from many who would otherwise not have participated, The participants of the study vary in terms of gender, degree course and year of study and so their comments may reasonably be taken as typical of a broad range of students.

## Results

Analysis of the interview and focus group data reveals that a number of factors may have contributed to the lack of uptake of mathematics support by failing students. Analysis of the responses from the survey interviews indicates that the factors that prevent students from accessing the support are similar to those that emerged from the analysis of the focus group and interview data. However, additional factors were also apparent that were not discussed by the participants from the focus groups and interviews, particularly amongst students from non-science departments. Some of these reasons will now be discussed in turn.

Analysis of the data reveals that many of these students did not use the Centre because they were unaware of its location. Six out of the 19 failing students (and a further 18 students from the remaining 73 students) claimed that they were unaware of where the Centre was located. Although advertising helps to promote an awareness of the MLSC, it may be that lack of knowledge of the location of this support is preventing some students from accessing its services.

In addition, the survey responses also revealed that many students lacked an awareness of what support facilities were available. Four failing students (and a further 17 out of the remaining 73 students) indicated that they had not used the Centre because they did not know enough about the MLSC. The responses given by these students suggest that many are not aware of the mathematics support services available or whether they are relevant to their individual needs.

Further analysis reveals that many students do not appear to be monitoring or directing their own learning and, consequently, students are unaware that support is needed. This was commented upon by eight out of the 19 failing students. From the focus group and interview data it appears that this is caused by two main factors, the first being a lack of motivation by the students. The data suggest that some students are failing to attend their timetabled tutorial sessions and, furthermore, they do not complete the work in their own time. Consequently these students are unaware of individual weaknesses that may require additional support. Furthermore, the data also suggest that some students are failing to manage their time effectively in order to cope with the demands and workload of their courses and, therefore, they often choose to work on their other modules rather than the mathematics module.

Some of the students who are failing to monitor and direct their own learning become overwhelmed by the amount of module material. From the data, two students in particular recalled that they had failed to grasp basic mathematical concepts and, as a result, the number of problems and their general lack of mathematical understanding increased. Consequently, these students felt that they had too many problems to address, and certainly too many to be solved in one visit to the MLSC. The data suggest that the students perceive the MLSC as a 'quick fix' to their problems as opposed to a long-term solution in supporting their lack of mathematical competency.

Data from the survey interviews revealed that students from non-science backgrounds do not perceive the MLSC as a place where they can obtain support (eight students commented upon this). Some students do not feel that the help provided is relevant to their courses, since they think of the MLSC as a place where students studying 'real maths' can obtain support. This is not the case, since the MLSC provides support for all students across the University.

An additional factor that prevented the students, who were questioned, from using the MLSC was feelings of embarrassment or intimidation. Nine out of the 19 failing students (and a further 10 out of the remaining 73 students) indicated that a reason why they had not accessed the support in the MLSC was because of a stigma associated with needing and asking for mathematics support. Students expressed that they felt particularly daunted by the prospect of asking for help from unfamiliar staff members and feared they would appear 'stupid' or would be mocked by the staff and their peers.

In some cases, students had their own personal reasons for not accessing the support via the MLSC. In particular, Student C, who had never used the Centre, had a preference to work on his weaknesses alone and did not feel comfortable obtaining help on a one-to-one basis.

## Summary and Conclusions

This paper has discussed a number of reasons why failing students are not engaging with mathematics support. These include a lack of awareness of the location and of the facilities available in the MLSC and a lack of awareness of the need for help. In particular, it appears that many students are failing to monitor and direct their own learning throughout the year, which means students are not aware of individual weaknesses until they come to prepare for the exam. In addition, some students feel embarrassed or intimidated about using the support for fear of being demoralised or mocked for their lack of mathematical competency.

To improve students' overall awareness of the MLSC, including its location and its facilities, it is suggested that an on-going advertising campaign is needed to promote the Centre. This could include distributing more leaflets and posters, particularly within the non-science departments, since this would be relatively easy to carry out. From the focus group and interviews it was suggested that more advertising should be carried out via e-mail contact. Students felt that by receiving regular updates from the MLSC informing them of the support available, they would be more likely to regard the Centre as an immediate point of help.

Additionally, the MLSC could be advertised via lecture recommendation. At present, lecturers are encouraged to advertise the Centre during the first week of term but it is possible that they do not regularly mention the MLSC throughout term time. It was suggested, by the students in this study, that mathematics lecturers should remind them of the MLSC every week and especially encourage those who appear to be struggling with mathematics.

Another course of action could be to specifically seek out those students who need mathematics support. Results from the focus group and interviews suggest that students could benefit from additional encouragement. This could involve targeting students who perform poorly in their mathematics coursework or exam. Such students would be given the opportunity to visit the Centre and talk to MLSC staff so that they become aware of the support facilities available to them and, in addition, this action would help them make the 'first step' of using the MLSC.

Finally, to help students overcome feelings of embarrassment or intimidation it is suggested that the MLSC could recruit members of staff whom the students are familiar with. Recruiting lecturers from other departments could be one method of action or recruiting postgraduates or finalist undergraduates, since students in this study suggested that they felt more comfortable asking their peers for help. However, departmental staff members will only be familiar to a handful of students who use the Centre and postgraduate students may lack the same breadth of knowledge and teaching skills as a lecturer.

Various suggestions to improve the uptake of support were made by the students. These include a more rigorous advertising campaign using leaflets, posters and e-mail contact, specifically targeting the weaker students. Students should also be encouraged to identify their weaknesses *and* to act upon these weaknesses.

Whilst there is no suggestion that the students answered the questions untruthfully, it may be that they have not analysed their own actions on a deep level. Students may have been unaware of the location of the MLSC but they did not take many steps to find it. Likewise students who were unaware of the nature of the support services available were not proactive in seeking to discover this information. It may be thought that if students were aware that the MLSC existed and also were aware that they had difficulties with mathematics and were in danger of failing their mathematics module then they would have taken steps to find out where the MLSC is located and whether or not it could help them. This analysis suggests that some of the reasons expressed by the students were only symptoms rather than the root cause. If this is correct, then actions to address the symptoms may not produce as great an improvement as may be expected.

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# Can't do maths, won't do maths – don't want help

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## Abstract

A sample of Stage 1 bioscience undergraduates, from seven UK universities, participated in a study to investigate the effect, if any, of a range of mathematics learning support resources on their basic mathematics skills. At the start of the 'experiment' students completed a basic mathematics test, comprising both abstract items and brief word problems set in biological contexts. Following the test, students were assigned randomly to one of the following learning resources: (i) prototype *biomathtutor* e-learning resources, (ii) a printed version of the *biomathtutor* resources, (iii) *mathtutor* e-learning resources. In addition, some students acted as a 'control' group, i.e. they were not assigned any additional maths learning resource. After a period of using the learning resource assigned to them, students (including the 'control' group) attempted a second test, the format of which was similar to that of the first test. When students' performances in the two tests were examined it was apparent that students found the tests challenging. In addition, students' test scripts were examined for their methods of calculation and, in particular, the types of errors and misconceptions they exhibited. In both tests students performed significantly better in the abstract test items than on the word problems. Students also exhibited a greater reluctance to attempt the word problems than the abstract test items. However, there was no significant difference in performance between any of the groups assigned different learning resources (including the control) in either the first or second test.

## Introduction

Universities and individual departments within them have adopted a diversity of strategies aimed at trying to address the mathematics problem evident amongst undergraduates embarking upon degree pathways across a variety of disciplines, with a view to better supporting their students' mathematics learning. These have included:

- offering pre-university summer courses for prospective entrants;
- designing and embedding foundation modules within degree pathways, where the curriculum is able to accommodate them;
- establishing tutor-led small group tutorial/workshop sessions at which the specific concerns and skills deficits of individual students may be addressed more directly;
- facilitating peer tutoring schemes;
- providing centrally-funded and located dedicated mathematics support centres or drop-in surgery facilities.

Many within the academic community, often in collaboration with other organizations, have designed a range of self-help, independent learning resources which may be either integrated within the curriculum and used within formal classes, or used by those students willing to engage in self-directed learning [1-4]. Such resources have included discipline-specific mathematics texts or workbooks, as well as computer-based learning

packages. Examples of the latter include *mathtutor* [3] (designed primarily, although not exclusively, for post-16 students going on to study mathematics or engineering) and *biomathtutor* [4] (designed specifically to support mathematics learning within the biosciences).

The aim of this study was to determine whether the use of a variety of mathematics learning resources by bioscience undergraduates might facilitate improvements in their basic mathematics skills.

## Methodology

Volunteer Stage 1 bioscience students from seven UK higher education institutions, representing both pre-1992 and post-1992 universities, participated in this study. All the students completed a twenty-item paper-based test, in class, which comprised both abstract basic mathematics questions and word problems set in biological contexts. The mathematics topics and concepts covered included volume, surface area, equations, fractions, powers of ten, and converting units of measurement. The following are examples of questions included in the tests:

- Find the value of  $x$  in the following linear equation:  $2(x + 9) = 3(2x - 3)$ .
- If each side of a cube is 6 cm in length, what is the surface area of the cube in  $\text{mm}^2$ ?
- In a scanning electron micrograph pollen grains have been magnified 500 times ( $\times 500$ ). Measuring one of the pollen grains with a ruler reveals a diameter of 3.2 cm. What is the real diameter of the pollen grain in  $\mu\text{m}$ ?
- Twenty small pieces of chocolate were placed a short distance from a mouse-hole. Over a period of 8 hours the mouse visited the supply of chocolate pieces four times, carrying away the same number of chocolate pieces on each visit. At the end of the eight-hour period twelve pieces of chocolate remained uncollected. How many pieces of chocolate did the mouse collect on each visit?

The students were allowed up to one hour to complete the test and were permitted the use of calculators.

Following the test, students were assigned to one of the 'experimental' groups summarised in Table 1.

'Experimental' groups	1st maths test (no. of students)	Additional mathematics learning support			2nd maths test (no. of students)
		Maths e-learning set in biological contexts	Maths learning set in biological contexts	Maths e-learning	
<i>biomathtutor</i> e-learning resource	P (125)	P			P (45)
<i>biomathtutor</i> workbook	P (87)		P		P (30)
<i>mathtutor</i>	P (36)			P	P (6)
control	P (39)	No additional mathematics learning support			P (8)
Total no. of students:	287				89

Table 1: Summary of the 'experimental' design and the numbers of students participating

Following a period of time (2-4 weeks) during which the students had access to their assigned mathematics learning resource (or no additional resource in the case of the control group), students attempted a second test, which was similar to the first test in terms of its mathematical content and the types of questions set. Students' performances in the two tests were compared. In addition, students' test scripts were examined for their methods of calculation and, in particular, the types of errors and misconceptions they exhibited.

## Results

Students appeared to find the two tests challenging, with 77% and 72% of students scoring 50% or less in the first and second tests respectively and with no student scoring higher than 90% in either test. There was no significant difference in test performance between students assigned to the different 'experimental' or control groups, either prior to assignment of the learning resources (i.e. in test 1) (ANOVA,  $F = 1.7$ ,  $p = 0.17$ ,  $N = 287$ ) or following the period of learning support provided (i.e. in test 2) ( $F = 1.4$ ,  $p = 0.25$ ,  $N = 89$ ). It is noteworthy that less than a third of those students who attempted the first test were actually willing to return for the second test (Table 1). This may have been, in part, a reaction to the students' poor performance in the first test, although students did not receive their marked scripts until the end of the 'experiment'. Maths anxious students in particular, of which there are a significant number amongst bioscience undergraduates, who found the first test challenging may have been 'put off' participating any further.

Further examination of the students' test scripts revealed that students performed significantly better in the abstract test items (mean score = 5.3,  $SD = 2.0$ ) than on the biological word problems (mean score = 2.6,  $SD = 1.7$ ;  $t = 15.40$ ,  $p = 0.001$ ,  $N = 89$ ) across both tests. Students also appeared more reluctant to attempt the word problems. For example, between 58% and 98% of students attempted the individual abstract items in the first test, while only 25% to 89% attempted individual word problems; this reluctance on the students' part may have reflected a lack of confidence, interest or understanding, a belief that familiarity with the biological topic was essential in order to attempt the question, and/or time constraints. Students encountered the greatest difficulties when it came to answering test items that required the calculation of volume and surface area, the conversion of units of measurement, fractions and powers of ten. In an attempt to try and understand better the nature of the mathematics skills deficit observed amongst bioscience undergraduates students, their answers were examined for the methods of calculation they adopted and, in particular, the types of errors they exhibited. In addition to errors that were probably due to carelessness, students exhibited a variety of procedural errors (e.g. when transforming equations) and misconceptions (e.g. when handling negative indices).

## Conclusions

Although the tests used in this study covered only basic mathematical concepts, which did not extend beyond those encountered by students at GCSE level, the majority of students found many of the test items too difficult to attempt successfully. The word problems set within biological contexts appeared to cause particular difficulties, with students' answers exhibiting a higher number of errors and students demonstrating a greater reluctance to even attempt many of the questions. Tobias drew attention to the fact that *'More than any other aspect of elementary arithmetic, except perhaps fractions, word problems cause panic among the math-anxious'* [5]. This finding has implications for mathematics support within the biosciences, where engaging students in problem-solving activities, which present the mathematics in context, is viewed as essential in order to encourage and motivate students to get to grips with the increasingly mathematical content of bioscience curricula. While many bioscience tutors acknowledge that a high proportion of their students are maths anxious, few may be aware of the effect that contextualizing the mathematics in word problems may have on such students; rather than assisting their students to overcome their maths anxieties, such contextualization may exacerbate the problem. Tutors will need to address their students' potential aversion to word problems through improving their problem-solving skills.

The difficulties students encountered with many of the basic mathematical concepts covered by the test items confirm the findings of previous studies which have used similar tests [6-7]. Of particular concern is the fact that many of the errors and misconceptions students exhibited reflect those commonly found amongst pre-GCSE pupils during their secondary level education [8]. Such inherent misconceptions may prove very difficult for tutors to overcome by the time students enter university.

Although the mathematics learning resources used in this study appeared not to result in any significant improvements in test performance, more extensive and reliable and valid data are required. However, few tutors and their students are often willing to engage in investigations of this nature. In addition, such educational 'field' studies are notoriously difficult to carry out successfully, since it can prove virtually impossible to control many of the intrinsic and extrinsic variables, e.g. students' backgrounds in mathematics, affective factors (maths anxiety, dyscalculia), failure of students to actually use the learning resource assigned to them or to use someone else's learning resource (in addition to or as a substitute for the one they were assigned), and the learning environment (including other sources of mathematics support available to different students). Faculty requirements for ethics approval, informed consent and participation on a voluntary basis can exacerbate the problems researchers encounter, resulting in low levels of student participation and/or high levels of student withdrawal from the investigation. The reluctance and apathy on the part of some students with regard to supporting such studies and/or using additional independent learning resources to support their mathematics learning is reflected in the following quotes: "don't want to do it", "have no time", "not interested", "can't be bothered".

While the reactions of students and their tutors towards mathematics learning resources such as **mathtutor** and **biomathtutor** are often very positive when evaluations using reaction questionnaires are carried out, unless those students who really need to use such resources are willing to do so, and unless such resources can be shown to deliver real improvements in mathematics skills competency is their development really worth the extensive investment of effort, time and money?

## References

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