

Problems for problem solving!

October 21, 2015

Group problem solving

A selection of problems for group work on problem solving

Birds and trains!

Two trains are approaching each other along a straight track. Each train is moving at a constant speed v . When the trains are a distance D apart a bird flies at speed $b > v$ directly from the front of one train to the other. Upon reaching the second train it immediately turns and flies back to the first. The process continues until the trains meet! How far has the bird flown at that time ?

Think about the assumptions you might make to tackle this problem.

Can you solve it ?

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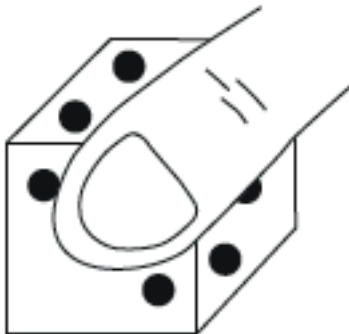
One strategy might be to start by putting some numbers in. Can you solve the problem if $D = 60$ miles, $v = 10$ mph and $b = 50$ mph ?

Cubes and acid!

A (uniform) metal cube is suspended from one of its corners and lowered into a vat of acid until exactly one half of it has been dissolved away. What is the shape of the new cross-section that has been created?

The partially obscured die

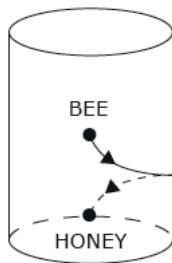
Here we see a conventional die partially obscured by a finger. Bearing in mind that dice are normally constructed so that opposite faces add up to seven, what is the number on the bottom face?



Bee in the honey pot

A bee lands on a cylindrical honey pot at a height that is exactly half way up the jar. Diametrically opposite the bee, a drop of honey has slid to the bottom of the pot.

Being mathematically minded and lazy, the bee crawls to this by the shortest route. The pot is 10 cm high and has a diameter of 24 cm. How far did the bee crawl?

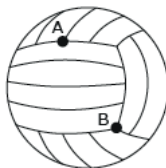


The football problem

Old-fashioned footballs used to be made out of a number n of equal rectangular strips as shown.

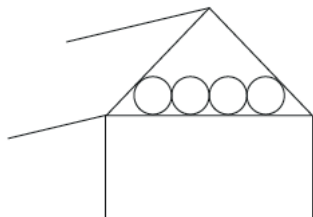
As you can see from the picture, the seams meet either at a T shaped junction (e.g. at the point A) of which there are t , or at a Y shaped junction (e.g. at B) of which there are y .

Calculate: $\frac{n \times t}{y}$.



Loft space

Four identical cylindrical tanks are needed to support my solar heating system. Luckily, they just fit exactly and snugly into the loft space as shown. If the roof space is 3m high and 8m wide, what is the diameter of the cylindrical tanks ?



Boys and girls

David has only two children and they are called Pat and Alex, which could equally be boys or girls names. In fact, Pat is a girl. What is the probability that Alex is a boy?

- ▶ a) 50%;
- ▶ b) Slightly less than 50%;
- ▶ c) Slightly more than 50%;
- ▶ d) Between 60% and 70%;
- ▶ e) Between 30% and 40%

Now, explain and justify your answer with colleagues near to you!

Sequences

Definition: A sequence a_n , $n = 1 \dots \infty$ is decreasing if and only if $\forall n \in \mathcal{N}$, $a_{n+1} \leq a_n$.

Definition: A sequence a_n , $n = 1 \dots \infty$ is increasing if and only if $\forall n \in \mathcal{N}$, $a_{n+1} \geq a_n$.

Give an example of a sequence (or state that this is impossible) that is both increasing and decreasing.

Roads

Build two straight roads that link the town, school and motorway so that the distance from town to school is as short as possible.



Integration

Imagine these problems are brought to you by students in your support centre. Think about how you would advise and assist each student.

$$\int_{-\pi}^{\pi} \sin 2x \, dx$$

$$\int \ln x \, dx$$

Quadratics

Explain what happens when the coefficients a , b and c are varied in the general form of a quadratic function

$$ax^2 + bx + c.$$

Estimating logarithms

Estimate the value of $\log_{10} 2$ to one decimal place.

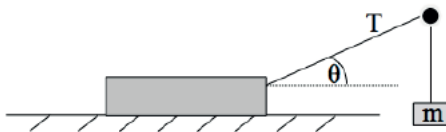
Provide a justification of your reasoning.

Special effects

A special effects studio wants to make a film of a 100m tower toppling to the ground using a 1m scale model. To make the footage realistic, the film needs to be slowed. By what factor should it be slowed down ?

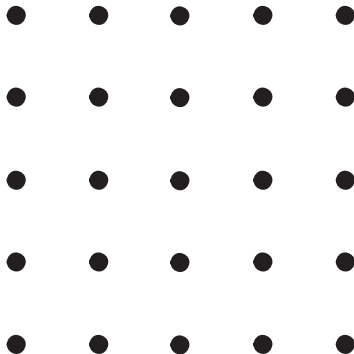
Dynamics

A block is being pulled across a rough horizontal surface as shown. The 'best' pulling angle, θ , is that which requires the least force to just move the block. Find what this is.



Lattice triangles

The infinite square dot-lattice is based on a unit square. In a set of all possible lattice triangles, that is, those triangles which have all three vertices at lattice points, what is the edge length of the smallest equilateral triangle ?



Quartering a rectangle

The teacher asked Daniel to use two straight lines to divide a rectangle into four equal parts and expected the picture on the left. He drew the one on the right. Is he correct or not ?

