Post-16 Mathematics Reforms: A level and Core Maths

- A level Mathematics and Further Mathematics

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Outline

1. Detail of the new A-level content and structure
2. Enhanced emphasis on understanding and problem solving
3. Decoupling AS and A-level mathematics
4. Emphasis on mechanics and statistics
5. Assessment objectives

Questions?

- Questions?

Aims

To provide modern A-levels that contain necessary material and that are also interesting to learn and teach and to serve a very wide user group (HE and work).

To:

1. build from GCSE
2. introduce calculus and its applications
3. emphasise how mathematical ideas are interconnected
4. show how mathematics can be applied to model situations mathematically
5. make sense of data
6. understand the physical world
7. solve problems in a variety of contexts
8. prepare students for further study and employment in a wide range of disciplines

Principles

1. content of the single mathematics A-level be fully prescribed - consistency, fundamentals
2. essential for a quality mathematics qualification at this level (currently choices reflect expertise - a challenge)
3. considerable detail with the recommended content
4. co-teaching of pure mathematics between the single A-level and AS-level further mathematics
5. AS-levels in mathematics and further mathematics should be supported and retained

Issues

1. mathematical thinking of the most able students is not developed
2. A vs. A* grades not for genuine mathematical ability - A* for demonstrating understanding and flair, not for doing routine calculations accurately - assessments developed accordingly
3. statistics - routine calculations vs. interpretation & understanding
4. some lack transferable skills
5. connected issues - assessment, delivery, structure and governance

Assessment

1. mainly test speed and accuracy rather than actual mathematical ability
2. examinations too short to allow for in-depth and searching questions
3. examinations have become repetitive and predictable
4. many questions are too highly 'scaffolded'
5. some more searching questions - problem solving/deeper understanding through demonstrating interpretation
6. sample assessment materials and examinations in order to make more definitive proposals
7. wider issues including the impact of linear A-levels, dangers to uptake, assessment, monitoring and implementation
8. outside the direct remit - solely the mathematical content

Core content

1. variety of mathematical concepts, methods and techniques - pure mathematics and applied, with overlap and interplay between them
2. need for better problem-solving skills: change in emphasis to problem solving, interpretation and testing understanding
3. drive assessment with less structured questions that test understanding and help to develop strategies for solving problems either in a purely mathematical or in an applications context

Overarching themes

Students to demonstrate the following overarching knowledge and skills:

1. OT1 Mathematical argument, language and proof
2. OT2 Mathematical problem solving
3. OT3 Mathematical modelling

Must be applied, along with associated mathematical thinking and understanding, across the whole of the detailed content set out below.

Integration in detail

Integration

	Content
H1	**[Know and use the Fundamental Theorem of Calculus]**
H2	**[Integrate x^n (excluding n = -1), and related sums, differences and constant multiples]** Integrate e^{kx}, $\frac{1}{x}$, $\sin kx$, $\cos kx$ and related sums, differences and constant multiples
H3	**[Evaluate definite integrals; use a definite integral to find the area under a curve]** and the area between two curves
H4	Understand and use integration as the limit of a sum
H5	Carry out simple cases of integration by substitution and integration by parts; understand these methods as the inverse processes of the chain and product rules respectively (Integration by substitution includes finding a suitable substitution and is limited to cases where one substitution will lead to a function which can be integrated; integration by parts includes more than one application of the method but excludes reduction formulae)
H6	Integrate using partial fractions that are linear in the denominator
H7	Evaluate the analytical solution of simple first order differential equations with separable variables, including finding particular solutions (Separation of variables may require factorisation involving a common factor)
H8	Interpret the solution of a differential equation in the context of solving a problem, including identifying limitations of the solution; includes links to kinematics

Statistical hypothesis testing in detail

Statistical hypothesis testing

	Content
O1	**[Understand and apply the language of statistical hypothesis testing, developed through a binomial model: null hypothesis, alternative hypothesis, significance level, test statistic, 1-tail test, 2-tail test, critical value, critical region, acceptance region, *p*-value]**; extend to correlation coefficients as measures of how close data points lie to a straight line and be able to interpret a given correlation coefficient using a given p-value or critical value (calculation of correlation coefficients is excluded)
O2	**[Conduct a statistical hypothesis test for the proportion in the binomial distribution and interpret the results in context]** **[Understand that a sample is being used to make an inference about the population and appreciate that the significance level is the probability of incorrectly rejecting the null hypothesis]**
O3	Conduct a statistical hypothesis test for the mean of a Normal distribution with known, given or assumed variance and interpret the results in context

Kinematics in detail

Kinematics

	Content
Q1	**[Understand and use the language of kinematics: position; displacement; distance travelled; velocity; speed; acceleration]**
Q2	**[Understand, use and interpret graphs in kinematics for motion in a straight line: displacement against time and interpretation of gradient; velocity against time and interpretation of gradient and area under the graph]**
Q3	**[Understand, use and derive the formulae for constant acceleration for motion in a straight line]**; extend to 2 dimensions using vectors
Q4	**[Use calculus in kinematics for motion in a straight line:** $v = \frac{\mathrm{d}r}{\mathrm{d}t},\ a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2 r}{\mathrm{d}t^2},\ r = \int v\ \mathrm{d}t,\ v = \int a\ \mathrm{d}t$ **]**; extend to 2 dimensions using vectors
Q5	Model motion under gravity in a vertical plane using vectors; projectiles

Matrices in detail

Matrices

	Content
C1	**[Add, subtract and multiply conformable matrices; multiply a matrix by a scalar]**
C2	**[Understand and use zero and identity matrices]**
C3	**[Use matrices to represent linear transformations in 2-D; successive transformations; single transformations in 3-D (3-D transformations confined to reflection in one of $x = 0$, $y = 0$, $z = 0$ or rotation about one of the coordinate axes) (knowledge of 3-D vectors is assumed)]**
C4	**[Find invariant points and lines for a linear transformation]**
C5	**[Calculate determinants of 2 x 2]** and 3 x 3 matrices and interpret as scale factors, including the effect on orientation
C6	**[Understand and use singular and non-singular matrices; properties of inverse matrices]** **[Calculate and use the inverse of non-singular 2 x 2 matrices]** and 3 x 3 matrices
C7	Solve three linear simultaneous equations in three variables by use of the inverse matrix
C8	Interpret geometrically the solution and failure of solution of three simultaneous linear equations

Problem-solving

Mathematical Problem Solving GCSE

1. develop mathematical knowledge, in part through solving problems and evaluating the outcomes, including multi-step problems
2. develop use of formal mathematical knowledge to interpret and solve problems
3. make and use connections between different parts of mathematics to solve problems
4. model situations mathematically and express the results using a range of formal mathematical representations, reflecting on how their solutions may have been affected by any modelling assumptions
5. select appropriate concepts, methods and techniques to apply to unfamiliar and non-routine problems; interpret their solution in the context of the given problem

Mathematical Problem Solving?

1. *Mathematical* problem solving - reducing the amount of 'scaffolding' and requiring students to apply mathematical knowledge with understanding and in other contexts

What should be labelled as problem-solving?

It is not necessary that every problem-solving question exhibit all the following attributes. At least one attribute should apply for a question to be identified as problem-solving:

1. Questions with little or no scaffolding: there is little help given, beyond a start and finish.
2. Questions which depend on multiple representations: such as use of a sketch or diagram as well as calculations.
3. The information is not given in mathematical form or in mathematical language; or the results must be interpreted in a real-world context.
4. Questions in which a choice of techniques (or statistical methods) could be used.
5. The solution requires understanding of the processes involved rather than just application of the techniques.
6. The question requires two or more mathematical processes.

Draft assessment objectives: Ofqual

1. AO1 Use and apply standard techniques 30-40% (35-45% for AS)
2. AO2 Reason, interpret and communicate mathematically 30-40% (and AS)
3. AO3 Solve problems within mathematics and in other contexts 30-40% (25-35% for AS)
4. AO2,3: 'Where problems require candidates to 'use and apply standard techniques' or to 'reason, interpret and communicate mathematically' a proportion of those marks should be attributed to the corresponding assessment objective.'

Draft assessment objective 1

	Assessment objective	Weighting	
		AS	A level
AO1	**Use and apply standard techniques** Students should be able to: • accurately recall facts, terminology, definitions and proofs • use and interpret notation correctly • accurately carry out routine procedures or set tasks requiring multi-step solutions	35-45%	30-40%

Draft assessment objective 2

	Assessment objective	Weighting	
		AS	A level
AO2	**Reason, interpret and communicate mathematically** Students should be able to: • independently construct a rigorous, non-standard proof or mathematical argument • construct extended chains of reasoning to achieve a given result • assess, critique and improve the validity of a mathematical argument, making deductions and inferences, finding and correcting errors in reasoning and evaluating evidence *Where problems require candidates to 'use and apply standard techniques' or to 'solve problems' independently a proportion of those marks should be attributed to the corresponding assessment objective*	30-40%	30-40%

Draft assessment objective 3

	Assessment objective	Weighting	
		AS	A level
AO3	**Solve problems within mathematics and in other contexts** Students should be able to: • translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes, identifying important features or variables and using appropriate techniques • make and use connections between different parts of mathematics • evaluate methods used and solutions obtained, recognising limitations and sources of error • construct, select and refine mathematical models • interpret the outcomes of a modelling process in real world terms and recognise the limitations of a model *Where problems require candidates to 'use and apply standard techniques' or to 'reason, interpret and communicate mathematically' a proportion of those marks should be attributed to the corresponding assessment objective*	25-35%	30-40%

Some questions

1. (i) Find the length of the radius of the largest circle which will fit inside a triangle with sides 3, 4, 5 cm.
   (ii) A circle C of radius 10 is drawn with centre at the origin. A second circle, lying in the first quadrant, touches C and the x and y axes, as in the (middle) figure below. Show that the length of the radius of the second circle is of the form $a + b\sqrt{2}$, where a and b are integers, stating the values of a and b.
2. Find the complete set of values of x for which $|x + 1| > 2|x - 1|$.
3. Sketch the graph of $y = |2x + 1| + |x - 2|$. Find the complete set of values of x for which $|2x + 1| + |x - 2| > 5$.
4. Solve the inequality $|50 - x^2| > 14$.
5. Find the value of the constant k so that the polynomial $P(x) \equiv x^2 + kx + 11$ has a remainder 3 when it is divided by $x - 2$. Prove that, with this value of k, $P(x)$ is positive for all real x, and sketch the curve $y = \dfrac{1}{P(x)}$.
6. Given that $a \neq 0$ and one root of the equation $ax^2 + bx + c = 0$ is twice the other root, prove that $2b^2 = 9ac$.

7. The polynomial $f(x)$ is given by $f(x) = x^3 - 4x + 15$.
   (a) Show that $x+3$ is a factor of $f(x)$. Show that $f(x)$ has no other real factors.
   (b) A curve has equation $y = x^4 - 8x^2 + 60x + 7$.
      (i) Find any stationary points of the curve. You may use your results from part (a).
      (ii) Determine, with a reason, whether the curve has a maximum point or a minimum point at each of its stationary points.
   (c) Draw a sketch of the curve $y = x^4 - 8x^2 + 60x + 7$, identifying any features indicated by earlier parts of this question.
8. p, q and r are positive constants, the roots of $px^2 + 2qx + r = 0$ are real and unequal, and the roots of and $qx^2 + 2rx + p = 0$ are real and unequal. Prove that the roots of $rx^2 + 2px + q = 0$ are not real.
9. Find the all solutions of the equation $4\cos x + 2\sin x = \sqrt{5}$ satisfying $0 \leq x \leq 360°$, giving your answers in degrees to three significant figures.
10. Sketch the graphs of $y = 4\sin x$ and $y = \sec x$ for $0 \leq x \leq 360°$. Find all solutions of the equation $4\sin x = \sec x$ satisfying $0 \leq x \leq 360°$, giving your answers in degrees.
11. BC is a chord of a circle with centre O and radius a. The mid-point of BC is M and MO is extended to A, where $OA = 2a$. Find, to the nearest degree, the angle BOM so that the triangle ABC has maximum area. Give your answer in degrees to three significant figures.

12. (i) Consider the curve with equation $y = \dfrac{\sin x}{5 - \cos x}$, where $0 \leq x \leq 2\pi$ in radians. By considering the values of $\cos x$ and $\sin x$ where $\frac{dy}{dx} = 0$, show that $-\frac{1}{12}\sqrt{6} \leq y \leq \frac{1}{12}\sqrt{6}$. Sketch the graph of y over the interval $0 \leq x \leq 2\pi$.
(ii) Show that the normal to the curve at the point where $x = \pi$ meets the y-axis at the point $(0, -6\pi)$.
(iii) Determine the area of the region bounded by the curve, the y-axis and this normal.
13. Evaluate $\displaystyle\int_0^{\pi} (\pi - x)\cos x \, \mathrm{d}x$.
14. Given that y is a function of x and is inversely proportional to $x + 1$, show that
$$(x+1)\frac{dy}{dx} + y = 0 \text{ and } y\frac{d^2y}{dx^2} = 2\left(\frac{dy}{dx}\right)^2.$$
15. A curve is defined by parametric equations $x = 8\mathrm{e}^{-2t} - 4,\ y = 2\mathrm{e}^{2t} + 4$. The tangent at the point where $t = \ln 2$ meets the x-axis in the point P. Find the coordinates of P.
16. A curve is given by parametric equations $x = 3t^3 + 4,\ y = t^2 - t + 1$. The tangent line at the point on the curve with $t = -1$ meets the curve again at the point P. Find the coordinates of P.

17 Two particles P and Q are at rest, each at a distance k metres from the origin, P being on the $x-$axis and Q on the $y-$axis. At the same instant each starts moving towards the origin, P with constant speed $k\,\mathrm{m\,s^{-1}}$ and Q with constant acceleration $k\,\mathrm{m\,s^{-2}}$. Find the rate at which the distance between P and Q is decreasing half a second later.

18 A rectangular block is of height h cm and has a square base of side x cm.

At a certain instant the volume of the block is increasing at the rate of 2% of its value per second and h is increasing at the rate of 3% of its value per second.

Is x increasing or decreasing at this instant and at what percentage rate if its value per second?

19 Given $y = \left(\dfrac{1+x}{1-x}\right)^n$, show that $(1-x^2)\dfrac{dy}{dx} = 2ny$.

20 Find the general solution of the differential equation $\dfrac{dy}{dx} + x\sin x\cos^2 y = 0$.

21 Given that $y = \dfrac{\sec x + \tan x}{\sec x - \tan x}$ show that $\dfrac{dy}{dx} = 2y\sec x$.

22. It is given that y is defined implicitly in terms of x by $x^2 - xy + y^2 = 1$. Determine $\frac{dy}{dx}$, simplifying as far as possible, and show that $\frac{d^2y}{dx^2} = \frac{6}{(x-2y)^3}$.

23. Express $f(x) = \frac{x^2 + x - 5}{(x-2)(x-1)^2}$ in partial fractions and hence evaluate $\int_3^4 f(x)dx$.

24. Determine the general solution of the equation $4\sin\theta = \sec\theta$.

25. Given that y is inversely proportional to $x+1$, show that $(x+1)\frac{dy}{dx} + y = 0$ and $y\frac{d^2y}{dx^2} = 2\left(\frac{dy}{dx}\right)^2$.

26. Show that the equation of the normal at $P(ct, c/t)$ to the rectangular hyperbola $xy = c^2$ is $t^3x - ty = c(t^4 - 1)$. The tangent at P meets the $x-$axis at A and the $y-$axis at B and the normal at P meets the $y-$axis at C. Show that the area of the triangle ABC is four times the area of the triangle AMP.

27. Given that $y = \tan^{-1}\left(\dfrac{1-x}{1+x}\right)$, determine $\dfrac{dy}{dx}$ in its simplest form.
28. Find all the values of x between 0° and 180° which satisfy the equation $\cos x = \cos 2x + \cos 4x$.
29. Evaluate $\cos^4 15^\circ + \sin^4 15^\circ$ *without* using a calculator.
30. Determine the ranges of values of k for which the equation $kx^2 + 6(k-2)x + 3(k+2) = 0$ has real unequal roots.
31. Given that $x + y = 1$ and $3^x = 2^y$ find the numerical value of 6^{xy}.
32. Sketch the curve whose equation is $y = 2x - 3x\,|x| + 2\,|x|$. Find the area of the region enclosed by the curve, the $x-$axis and the lines $x = -1$ and $x = +1$.
33. Prove that $(\cos\theta + \sin\theta)^4 \equiv \frac{3}{2} + 2\sin 2\theta - \frac{1}{2}\cos 4\theta$.
34. Sketch the graph of $y = x(x-1)(x-3)$. A is the point $(-1,-8)$ on this graph. Write down the equation of the line through A which has a gradient m, and find for what range of values of m the line cuts the graph again in real points P and Q. Show that when it does the mid-point of PQ lies on a fixed line parallel to the $y-$axis, and find the equation of this line.
35. Find the possible range of values of $y = \dfrac{x(x+1)}{x-1}$ for real values of x.

36. Given that $\cos 2\alpha = \dfrac{2\cos 2\beta - 1}{2 - \cos 2\beta}$, prove that $\tan^2\alpha = 3\tan^2\beta$.

37. The function $f(x)$ is defined for all real numbers and has the following properties, valid for all x and y:

    (A) $f(x+y) = f(x)f(y)$
    (B) $\dfrac{df}{dx} = f(x)$
    (C) $f(x) > 0$

    Let $a = f(1)$.

    (i) Show that $f(0) = 1$.
    (ii) Let $I = \int_0^1 f(x)dx$. Show that $I = a - 1$.
    (iii) The trapezium rule with n steps is used to produce an estimate I_n for the integral I. Show that

38.
    1. Given that $2f(x) - f(-x) = 1 + x\int_{-1}^{1} f(t)dt$, prove that $\int_{-1}^{1} f(t)dt = 2$.
    2. Given that $f(x) + f(-x) = 1 + g(x)\int_{-1}^{1} f(t)dt$, prove that $g(x)$ is an even function, and calculate $\int_{-1}^{1} f(t)dt$ in the case $g(x) = x^2$.

[core:maths] support programme

A FRESH APPROACH TO A NEW QUALIFICATION

CfBT Education Trust

Post-16 Mathematics Reforms: A-level and Core Maths

- Core Maths

Paul Glaister

University of Reading
(CMSP, DfE)

CETL-MSOR Conference ‘15

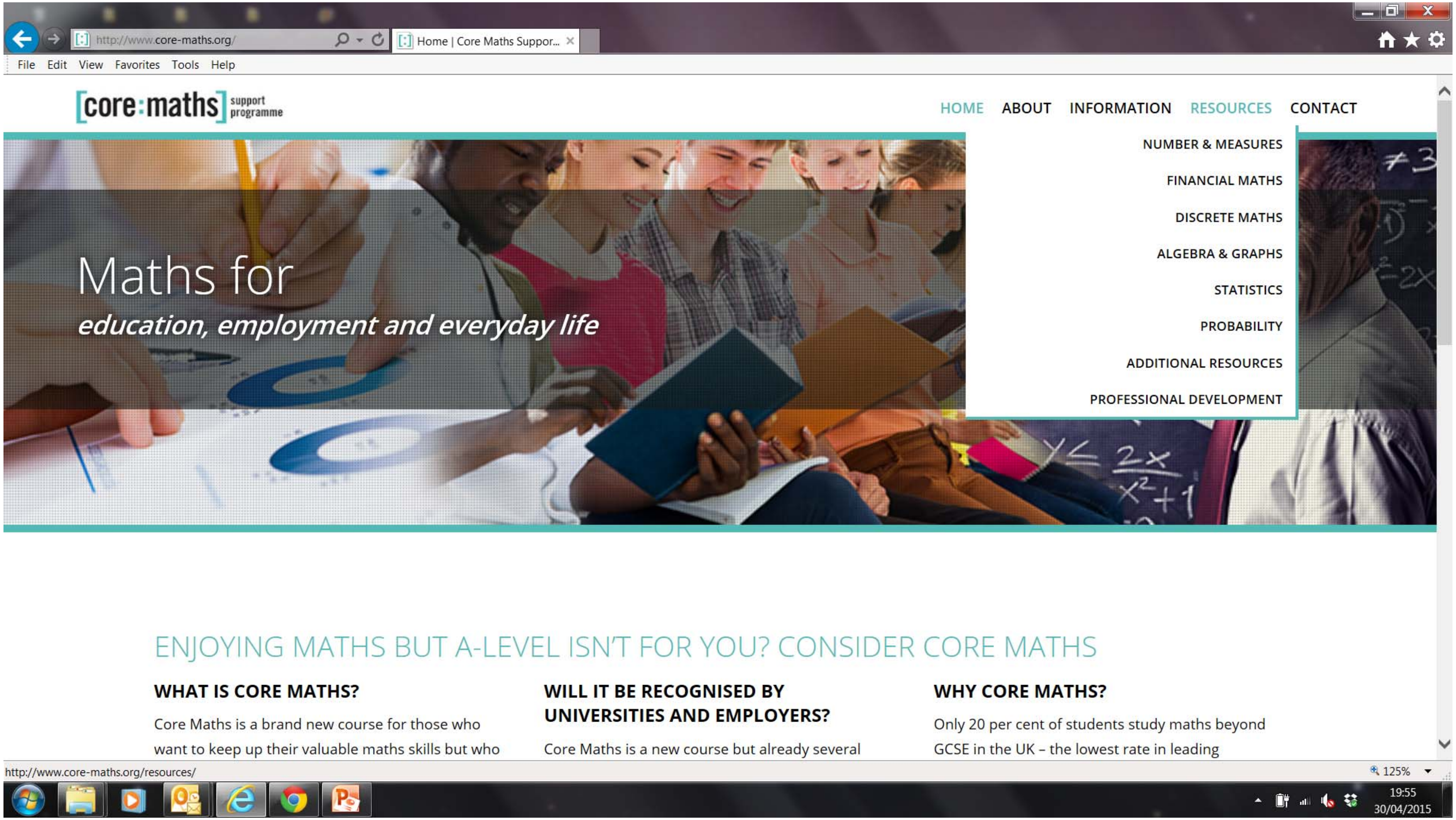
website: http://www.core-maths.org
email: p.glaister@reading.ac.uk

This session will seek to answer the questions:

- ***who*** **is Core Maths for**
- ***why*** **was Core Maths introduced**
- ***what*** **is Core Maths for**
- ***where*** **does Core Maths fit into the HE landscape**
- ***when*** **will Core Maths be common currency**

- **Questions?**

CMSP website



Background to Core Maths

June 2011 Michael Gove’s address to the Royal Society

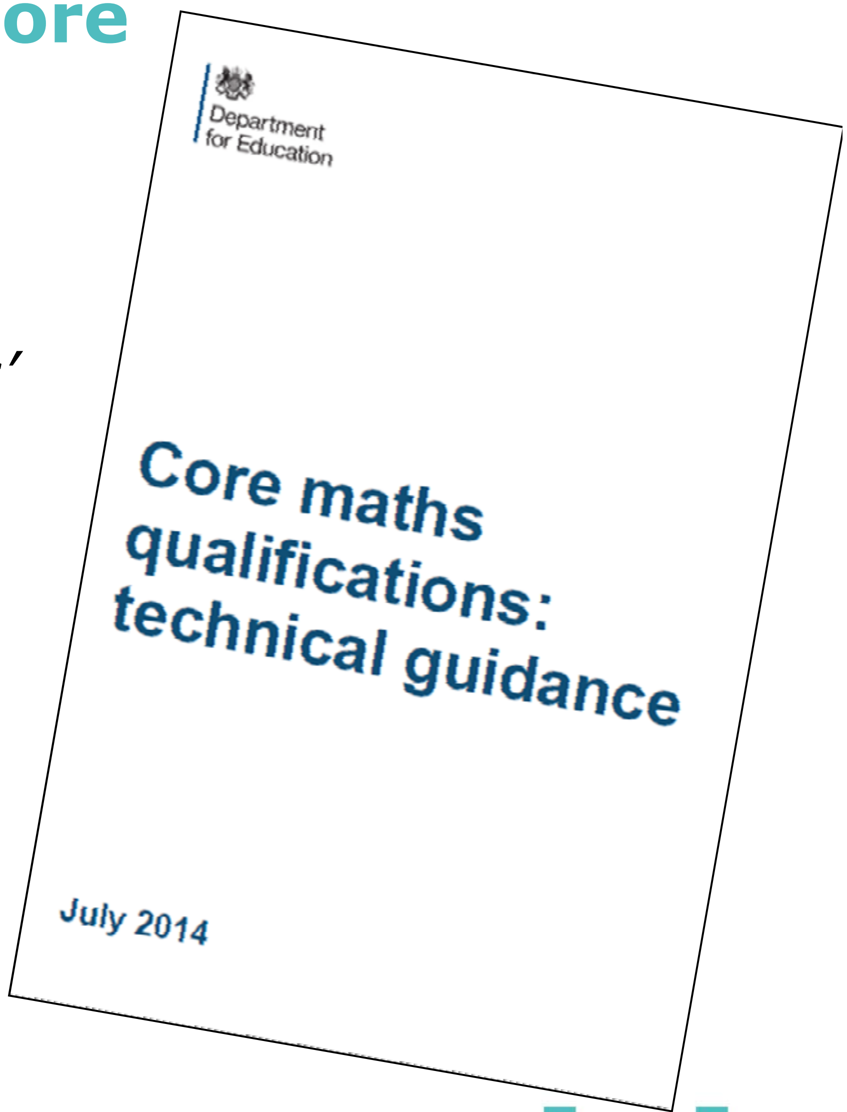
‘ … we should set a new goal for the education system so that within a decade the vast majority of pupils are studying maths right through to the age of 18.’

- **Purpose of Core Maths**

Technical guidance for Core Maths qualifications

Purpose

- *Core Maths qualifications should consolidate and build on students' mathematical understanding and develop further mathematical understanding and skills in the application of maths to authentic problems, thereby offering progression from GCSE mathematics.*
- *Qualifications should provide a sound basis for the mathematical demands that students will face at university and within employment across a broad range of academic, professional and technical fields.*


Department for Education

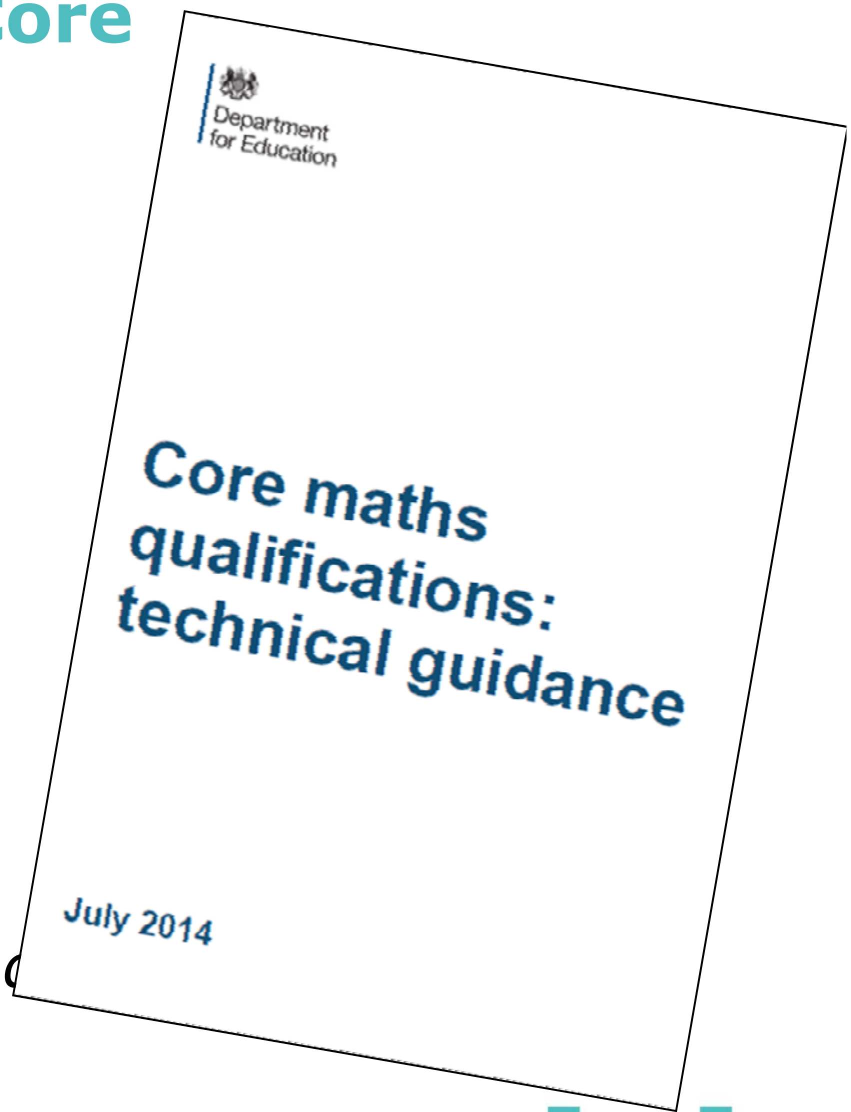
Core maths qualifications: technical guidance

July 2014

Technical guidance for Core Maths qualifications

Objectives

1. *Deepen competence in the selection and use of mathematical methods and techniques.*
2. *Develop confidence in representing and analysing authentic situations mathematically and in applying mathematics to address related questions and issues.*
3. *Build skills in mathematical thinking, reasoning and communication.*



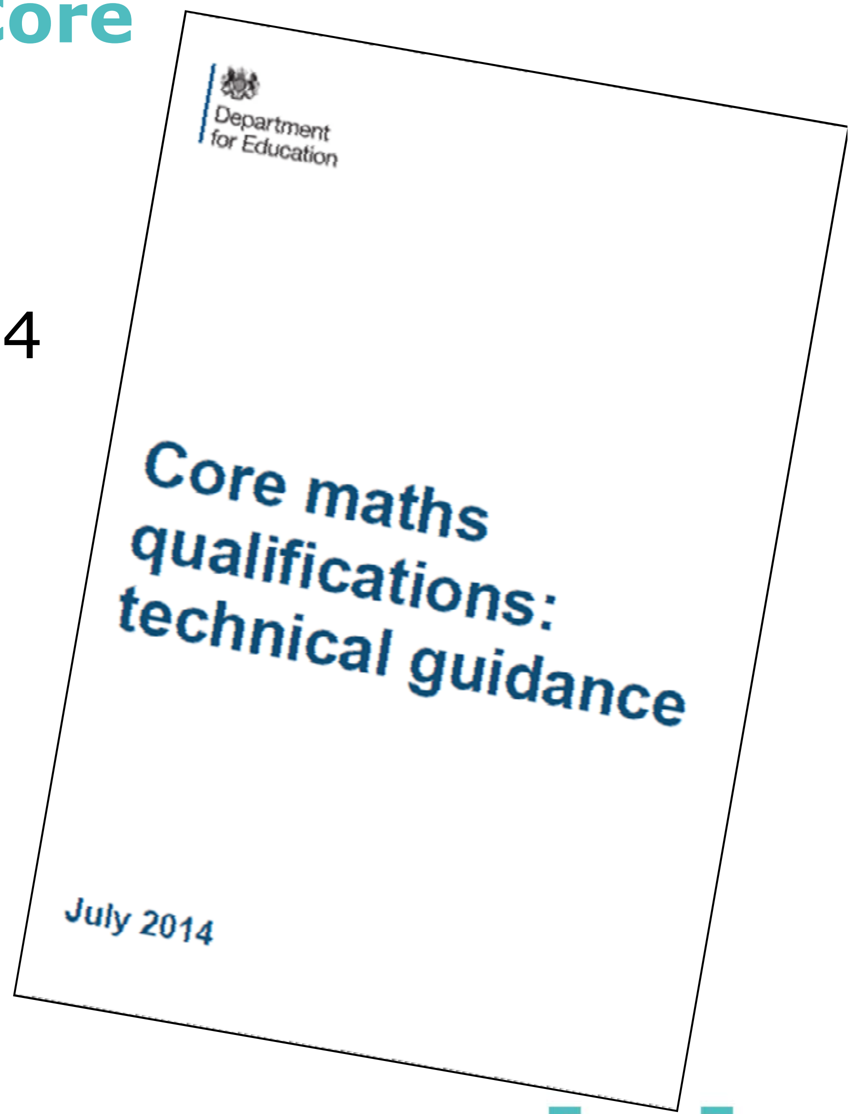
Department for Education

Core maths qualifications: technical guidance

July 2014

Technical guidance for Core Maths qualifications

- Published July 2014 after consultation from April 2014
- 180 guided learning hours
- Up to 80% Higher level content from the 2015 GCSE
- At least 20% level 3 content
- Problem solving approaches
- Terminal assessment
- Not an AS but same UCAS tariff as AS


Department for Education

Core maths qualifications: technical guidance

July 2014

Launch of new high-quality post-16 maths qualifications

- New high-quality maths qualifications, which teach pupils how to use and apply maths in real situations, are designed to encourage thousands more pupils to continue studying maths beyond age 16, School Reform Minister Nick Gibb announced today (5 December 2014).

Core Maths qualifications:

- Mathematical Studies (AQA) – three options
- Using and Applying Mathematics (City & Guilds)
- Mathematics in Context (Pearson/Edexcel)
- Mathematics for Work and Life (eduqas/WJEC)
- Quantitative Reasoning/Quantitative Problem Solving (OCR/MEI)

- ALL BRANDED Core Maths

- HE need for Core Maths

HEA Transitions report: mathematical needs of students undertaking undergraduate studies in various disciplines (July 2014)

Investigated the mathematical and statistical requirements of:

- Business and Management,
- Chemistry,
- Computing,
- Economics,
- Geography,
- Sociology and
- Psychology

HEA Transitions report: mathematical needs of students undertaking undergraduate studies in various disciplines (July 2014)

- All require Mathematics and/or Statistics to some extent
- 85,000 students are admitted into university each year to study the seven disciplines in the project

HEA Transitions report: mathematical needs of students undertaking undergraduate studies in various disciplines (July 2014)

- Other disciplines have similar mathematical and statistical needs, including biological sciences, medicine and dentistry, architecture, building and planning, and various technology degrees.
- The number of students affected is of the order of 200,000 pa.

HEA Transitions report - findings

- The number of students entering the disciplines with an A or AS-level in Mathematics has increased in recent years but has probably reached a limit.
- Many students arrive at university with unrealistic expectations of the mathematical and statistical demands of their subjects.
- Lack of confidence and anxiety about Mathematics/Statistics are problems for many students.

HE – requirements: skills

- Know, understand and use existing GCSE material with confidence
- Solve problems in a variety of contexts
- Make logical and reasoned decisions
- Communicate
- Generalise
- Interpret
- Make deductions
- Use technology
- Modelling
- Algebraic manipulation

HE – requirements: topics

- Fractions, ratios, percentages, and decimals
- Inequalities
- Graphs
- Algebra
- Probability
- Correlation
- Hypothesis testing
- Summary statistics

HE - now

How do they cope?

- Maths and stats courses
- Maths and stats support centres

Why don't they currently ask for maths?

- Supply
- Competition

Core Maths topics (sample)

- Analysis of data
- Maths for personal finance
- Estimation
- Critical analysis of given data and models (including spreadsheets and tabular data)
- The normal distribution
- Probabilities and estimation
- Correlation and regression
- Critical path analysis
- Expectation
- Cost benefit analysis
- Graphical methods
- Rates of change
- Exponential functions
- (STATISTICAL) PROBLEM SOLVING
- MODELLING

Core Maths - HE

Key message:

- better prepared students & employability

Key issues:

- Supply vs. demand
- Competition
- Selectors/recruiters

Key goal:

- Requirement?
- 'particularly welcome Core Maths' or 'AS/A-level'

Some endorsements

Loughborough University

We support the ethos behind the introduction of this new qualification in that it may be beneficial to a range of degree subjects that do not generally ask for A Level Mathematics but where enhanced numerical or statistical skills may be helpful. For this reason, we encourage applicants to consider taking this qualification where practical.

University of Southampton

The University fully supports the principles behind the introduction of the new Core Mathematics qualifications and believes they could be beneficial to students considering making an application to a range of degree subjects in the social sciences, business, and health sciences, for example.

[c:m]

- **Answers √**
- **More questions?**

Thank you