Mathematics after 16: the state of play, challenges and ways ahead

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www.nuffieldfoundation.org

About this report

This report is based on a presentation given by the author, Josh Hillman, on 17 March at the first Q-Step conference, *Counting them in: quantitative social science and the links between secondary and higher education*. Other presentations from the day are available at www.nuffieldfoundation.org/q-step.

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Foreword

The starting point for this report was a presentation given by its author, Josh Hillman, at a Q-Step conference convened by the Nuffield Foundation to discuss quantitative skills in the social sciences, and the links between secondary and higher education. This context is a useful reminder of one of the most important factors in the debate about post-16 mathematics: namely that this is not a 'maths problem', nor one that affects only STEM subjects. It is an issue that cuts across different subjects and different stages of education. It is about ensuring that all young people have the quantitative skills necessary for further and higher education, for work, and for being informed citizens.

The Foundation's 2010 report, *Is the UK an outlier?,* showed that England, Wales and Northern Ireland have lower participation rates in post-16 maths than comparable OECD countries. Our follow-up report, *Towards universal participation* (2013), showed that countries with high levels of participation all offered more than one high-status option for post-16 students to continue their mathematics education. The report recommended the development of a new mathematics qualification aimed at students for whom the traditional Mathematics AS/A level may not be suitable.

A lot has changed since the publication of these reports. The government has made a commitment to a goal of universal participation in post-16 mathematics, and is now introducing a new Core Maths qualification outside of the AS/A level structure. While we welcome the general commitment wholeheartedly, this report sets out some concerns about the timetable for change, and about the need to take all the relevant evidence into account. This includes information on current participation patterns, as well as lessons learned from previous attempts to increase participation. It is also important to consider the broader context, and to ensure that reforms to GCSEs and A levels support rather than undermine efforts to increase participation.

Our aim in publishing this report is to bring together the most relevant evidence related to post-16 mathematics participation and to offer a constructive contribution to the reform agenda. The report outlines the complex factors that need to be considered as part of longer-term efforts to increase engagement with quantitative skills. It also sets out the main challenges to the goal of universal participation and presents a number of ways in which they might be addressed.

Our ambition is to see teaching and learning of quantitative skills embedded across all post-16 education provision via complementary pathways. In decades to come, when it is asked whether the UK is an outlier, we want the answer again to be a resounding yes, but this time for us to be at other end of the scale.

David Rhind

Professor David Rhind
Chair of Trustees

Introduction

This report focuses on the crucial period between GCSEs and entrance to higher education, and how mathematical, statistical and quantitative skills develop during this period. The importance of these skills (for which we use 'maths' as shorthand) for individuals, the economy and for society as a whole is taken as read.[1]

The report has three aims.

1. Outline the current state of play in post-16 mathematics and the explanations for it.
2. Describe a number of challenges in this area and assess how government policy attempts to address them.
3. Pose a number of ways in which these challenges might be addressed, including through work funded and undertaken by the Nuffield Foundation.

The focus of the report is the education system in England, Wales and Northern Ireland. The situation in Scotland is different, and is touched on only briefly. I do not deal directly with the mathematical content of vocational tracks, which would require another report in itself. However, it is important to note that those taking vocational routes who have achieved a good grade in GCSE Mathematics are being encouraged to take qualifications common to academic tracks.

1. The state of play

"It is vain to conceal the melancholy truth. We are fast dropping behind. In mathematics we have long since drawn the rein, and given over a hopeless race."

Charles Babbage, *Reflections on the Decline of Science in England, and on Some of its Causes*, 1830.

Concern that the UK may be falling behind in the global race to improve the maths education of its young people is far from new, as this quotation from Babbage shows. One hundred and eighty years later, the Nuffield Foundation published *Is the UK an outlier? An international comparison of upper secondary mathematics*, which presented findings from a study of participation in post-16 maths in 24 OECD countries. That study, led by Professor Jeremy Hodgen at King's College London, found that fewer than one in five students in England, Wales and Northern Ireland study any kind of maths after GCSE, the lowest levels of participation in the 24 countries surveyed. Levels of participation are higher in Scotland, where just under half of students study maths after S4, but still below the average of the countries studied. By contrast, in 18 of the 24 countries more than half study maths post-16, while in eight of them there is near universal take-up, with participation rates at or above 95%. However, it is important to note that high levels of participation reflect the fact that mathematics is a compulsory subject post-16 in these countries, unlike in the UK.

[1] *Mathematical Needs: Mathematics in the workplace and in Higher Education* (London: ACME, 2011).

England, Wales and Northern Ireland also have some of the lowest participation rates in advanced maths, with approximately 13% of 16- to 18-year-olds taking A level Mathematics in England,[2] 11% in Wales and 15% in Northern Ireland. Again, on this measure Scotland does better, lying in the mid-range of countries, with take-up of advanced maths at around 23%.

The proportions of post-16 students studying any maths, and studying advanced maths are shown in **Table 1** and **Table 2**.

Table 1: Proportion of post-16 students studying any maths[3]

All (95-100%)	Czech Republic, Estonia, Finland, Japan, Korea, Russia, Sweden, Taiwan
Most (81-94%)	Canada (BC), France, Germany, Hungary, Ireland, USA (Mass)
Many (51-80%)	Australia (NSW), Netherlands, New Zealand, Singapore
Some (21-50%)	Hong Kong, Scotland, Spain
Few (6-20%)	England, Wales, Northern Ireland

Note: The base for the percentages is the number of students in post-16 (or 'upper secondary') education or training.

Table 2: Proportion of post-16 students studying advanced maths[4]

High (31-100%)	Japan, Korea, New Zealand, Singapore, Taiwan
Medium (16-30%)	Australia (NSW), Estonia, Finland, France, Hong Kong, Scotland, Sweden, USA (Mass)
Low (0-15%)	England, Germany, Ireland, Netherlands, Northern Ireland, Russia, Spain

Note: The base for the percentages is the number of students in post-16 (or 'upper secondary') education or training. Data on participation in advanced maths was insufficient in Canada (BC), Czech Republic and Hungary.

2 This represents approximately 11% of the age cohort (including those not in education and training).
3 Jeremy Hodgen and others, *Is the UK an outlier? An international comparison of upper secondary mathematics education* (London: Nuffield Foundation, 2010), p. 38.
4 Hodgen and others, p. 38.

It is important to note that this situation is dynamic, as all governments, wherever their countries sit in the table, are developing reforms to curricula and qualifications with increasing participation as an aim. In other words, participation rates in other countries seem likely to improve over time, and this is a relevant factor if we view it as important to keep up with other nations. The scale of the change needed in the UK is enormous.

The structure of A level Mathematics and Further Mathematics

What lies behind the UK's comparably poor participation rates? In England, Wales and Northern Ireland, with some small exceptions, post-16 maths, for those bound for university, means AS and A level. For some, it will include Further Mathematics. It is therefore important to set out the complex structure of these qualifications before considering trends in the number of young people studying for them.

AS/A level Mathematics and Further Mathematics are currently all incorporated in a single structure in which the modular route taken determines the qualification, although this modularity will be removed from 2016. There are many modules to choose from, though some are compulsory and students' choices are often constrained, or even dictated, by what is available in schools or colleges.

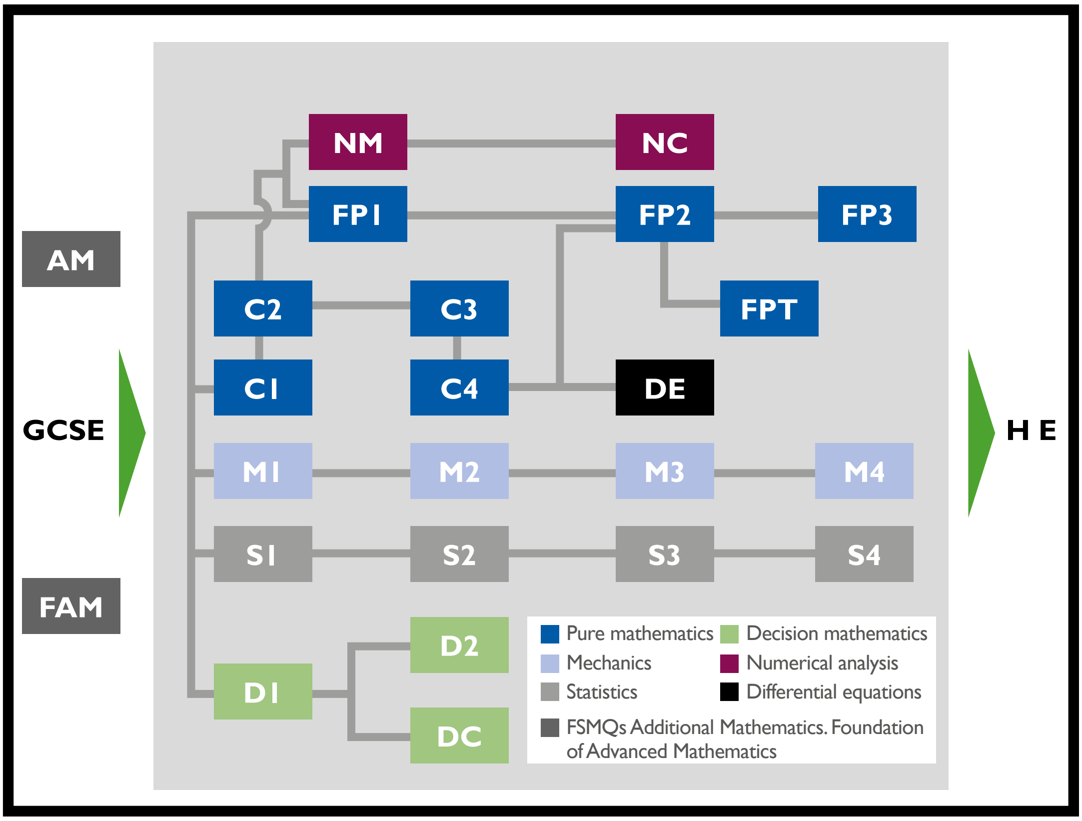
Figure 1 shows the modules available in the current OCR (MEI) Mathematics and Further Mathematics A levels. Similar structures apply for other awarding organisations, and as discussed later, all will change in 2016.

Students start with AS Mathematics. This consists of the two AS units in Compulsory Pure Mathematics, C1 and C2, together with one applied unit in either Mechanics (M1), Statistics (S1) or Decision Mathematics (D1). To complete the full A level, students take three more units, C3, C4 and another applied unit. The applied unit may be in the same strand as that taken for the AS or from a different strand.

Further Mathematics is only taken by students who are also taking Mathematics. AS Further Mathematics consists of Further Pure 1 (FP1) and two other units. To complete the A level in Further Mathematics, students take another three units.[5]

[5] *MEI Structured Mathematics specification* (MEI/OCR, 2013). Available: http://www.ocr.org.uk/Images/75811-specification.pdf [accessed 12 June 2014].

Figure 1: OCR Mathematics (MEI) specification 2013



Trends in participation rates

So England, Wales and Northern Ireland have comparatively low rates of participation in maths at post-16. And for those who do continue to study maths, AS/A level mathematics is the dominant post-16 qualification. But what do we know about trends in student take-up: is post-16 participation improving or not?

Figure 2 shows a sharp slump in the take-up of A level Mathematics shortly after the turn of the century. This was associated with the Curriculum 2000 reforms, which introduced a universally modular structure for the A level with a new AS qualification as part of that. There was a surge in the number of students dropping mathematics in 2002, and it took six years for A level entry numbers to recover. But since 2006 they have displayed a steady upward trend. By 2009, entries had exceeded their 1992 level, and by 2013, they were 22% higher. The pace of increase in the last five years has been even faster for AS Mathematics, suggesting some broadening of take-up, as more young people at least try the subject after GCSE. This has provided a valuable boost to post-16 participation.

Some of the increase in student numbers taking AS and A level Mathematics has been driven by a rise in the size of the cohort. However, unlike the position in the sciences, there has also been an increase in the proportion of the cohort taking maths, particularly since 2006.

This means that maths is now not far behind English as the most popular A level, while it is the most popular at AS. However, this is still a small minority of the cohort: even the improved number of 88,060[6] UK participants in A level Mathematics in 2013 should be considered in the context that national year groups contain over 600,000[7] young people.

[6] Joint Council for Qualifications (JCQ), *Provisional GCE A Level Results – June 2013 (All UK Candidates* (London: JCQ, 2013). Available: http://www.jcq.org.uk/examination-results/a-levels/a-as-and-aea-results-summer-2013 [accessed 20 May 2014].

[7] Department for Education (DfE), *Statistical First Release: Schools, Pupils, and their Characteristics, January 2011* (London: DfE, 2011). Available: https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/219064/main_20text_20sfr122011.pdf [accessed 20 May 2014].

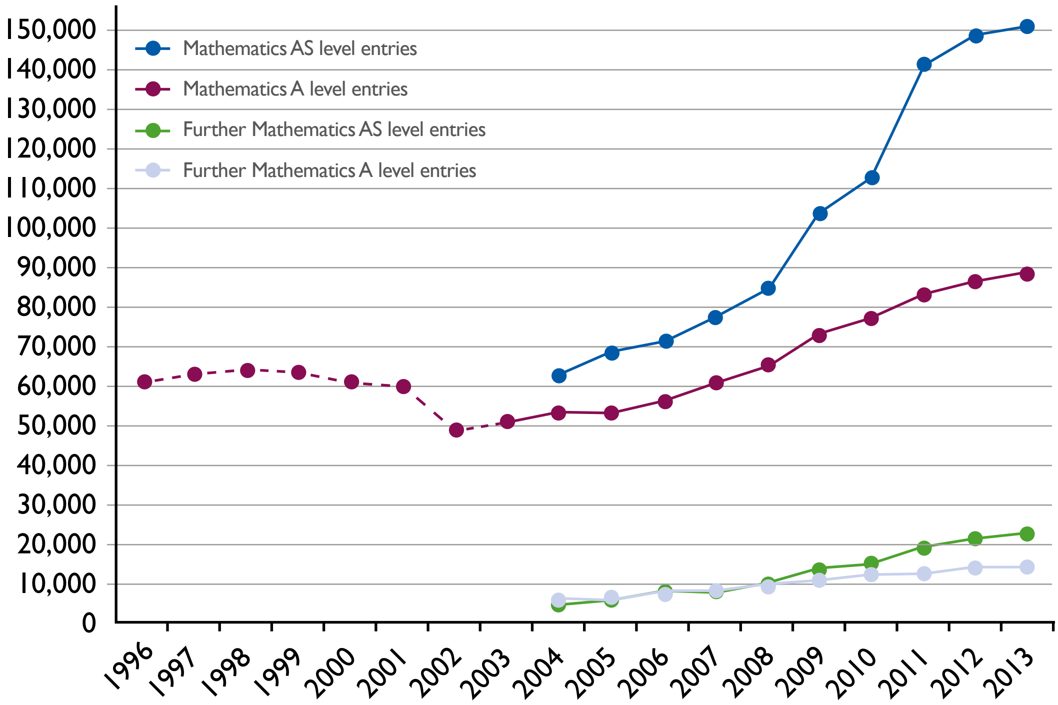


Figure 2: Mathematics AS and A level entries (UK)[8]

Note: Data prior to 2003 is represented by a dotted line as it involves estimation. Prior to 2003, Mathematics and Further Mathematics A level numbers were reported collectively. The disaggregated estimates for pre-2003 A level numbers used here are based on related proportions for England.[9]

2. Why is the UK an outlier? Six challenges

I have set out six challenges facing current attempts at reform. These are considered within the context of the government's stated aspiration for universal participation in mathematics for 16- to 18-year-olds. In a speech at the Royal Society, Michael Gove, the Secretary of State, said: "We should set a new goal so that within a decade the vast majority of pupils are studying maths right through to the age of 18."[10] This statement, which came a few months after the publication of the Foundation's *Is the UK an outlier?* report, is an important step forward. Yet it is important to look beyond the rhetoric and to consider how current reforms to A level and GCSE, and the introduction of the new core maths qualification(s) might affect the goal of universal participation in maths after age 16.

In setting out the explanations and challenges, I describe and briefly assess these current reforms. However, it not an easy task, as they have been emerging from slightly Byzantine processes within the Department for Education and its agencies, with reform often in flux or faltering, and neither wholly transparent nor clearly communicated except in fragments.

[8] Data from the Joint Council of Qualifications. Figures from 2001-2013 from : http://www.jcq.org.uk/examination-results/a-levels [accessed 20 May 2014]. Cumulative Mathematics and Further Mathematics figures from 1996-2003 from Roger Porkess and Stephen Lee, *A Level Mathematics & Further Mathematics: An investigation into the reasons for increased uptake in 2009* (Trowbridge: MEI), 2009 p. 11 (http://www.mei.org.uk/files/pdf/A_Level_uptake.pdf) [accessed 20 May 2014].

[9] Jeremy Hodgen, Rachel Marks and David Pepper, *Country profile: England.* (London: Nuffield Foundation, 2013), p. 16. Available: http://www.nuffieldfoundation.org/sites/default/files/files/ENGLAND%20country%20profilev_FINAL.pdf [accessed 20 May 2014].

[10] *Michael Gove speaks to the Royal Society on maths and science*, 29 June 2011. Available: https://www.gov.uk/government/speeches/michael-gove-speaks-to-the-royal-society-on-maths-and-science [accessed 20 May 2014].

Challenge 1: the wider 14-to-19 qualification framework and early specialisation

The policy landscape in recent decades is littered with strategic attempts to broaden the post-16 curriculum. At least six major reviews can be identified during this period, all following a similar theme. However, none of these have effected significant change, and the dominant route for those who wish to continue to study maths – the A level, with its relatively narrow candidate group – has retained its pre-eminence despite continuing concerns about the lack of options for those not pursuing this pathway. This suggests a significant increase in participation will continue to be elusive without a shift in political will and consensus across parties.

Attempts to broaden the post-16 curriculum: key reviews and reports

A British 'Baccalauréat': Ending the Division between Education and Training (1990), an influential paper written by a team of researchers including the future schools minister, David Miliband, and published by the Institute for Public Policy Research (IPPR). [11]

The National Commission on Education, set up in 1991 under the auspices of the then British Association for the Advancement of Science, which recommended a General Education Diploma.[12]

The Dearing Review of 16-to-19 Qualifications (1996), which recommended a national framework of qualifications embracing the academic and the vocational.[13]

The Tomlinson Report (2004), which put forward a unified structure subsuming GCSE, A level and vocational courses in a broad diploma system. [14]

The Nuffield Review of 14-19 Education and Training (2009), which called for a "broader vision of education", including a "unified system of qualifications".[15]

The Skills Commission's *One System, Many Pathways* report on 14-19 education (2013), which argued for improving non-academic routes.[16]

Challenge 2: Issues around A level Mathematics

Are there characteristics of A level Mathematics that themselves contribute to low participation? Regardless of the importance of promoting and improving non-A level routes, the Nuffield Foundation believes the dominant academic route itself should be re-evaluated. We are therefore funding a wide-ranging study, using mainly quantitative methods, to explore its value in all senses of the word. The study is being led by Professor Andrew Noyes at the University of Nottingham, and is addressing the following questions:

[11] David Finegold and others, 'A British 'Baccalauréate' - Ending the Division Between Education and Training', *Education and Training Paper No 1* (London: IPPR, 1990).
[12] *Learning to Succeed:* The Report of the Paul Hamlyn Foundation Commission on Education, (London: Heinemann, 1993).
[13] *Review of Qualifications for 16-19 year olds: full report of the Dearing Review* (Hayes: SCAA, 1996).
[14] *14-19 Curriculum and Qualifications Reform: Final Report of the Working Group on 14-19 Reform* (London: DfES, 2004).
[15] Richard Pring and others, *Education for All: The Future of Education and Training for 14-19 Year-Olds* (London: Routledge, 2009).
[16] *One System, Many Pathways: Forging Consensus on 14-19 Education and Training* (London: Skills Commission/Policy Connect, 2013).

- What is the relationship of Mathematics A level to degree-level study, whether in maths or in other subjects?
- Who is studying A level Mathematics, in terms of students' gender, their socio-economic backgrounds and the types of schools and colleges they attend?
- What do young people themselves think of A level Mathematics content and purposes?
- What is the economic return to participation and achievement in A level Mathematics?

Current Mathematics A level reform

The reforms currently underway are complicated and at risk of being poorly co-ordinated. In 2013, Ofqual identified A level Mathematics and Further Mathematics as needing "significant change".[17] This overhaul is running to a later timetable than most other A levels, with first teaching set for September 2016, but still leaves little scope for co-ordination with other subjects. The main reasons for this delay are not changes to content or pedagogy but the decision to replace the modular structure with a linear one and to accommodate terminal exams, as well as to try and ensure the take-up of Further Mathematics is not harmed by the change in structure.

From 2016, AS Maths will only be available as a stand-alone course, and its grade will not contribute to that of the full A level. Yet it appears that the A level's modular structure has facilitated the growth in take-up over the past eight years, partly because students have been able to build qualifications, and their own confidence, module by module. So they can stop at AS level and achieve a good grade in a recognised and valued qualification. The new reforms are an attempt to curb a 'resit culture'. But there is a danger that a move to a less flexible model may result in a decrease in student numbers. For example, students who are not confident about committing to A level Mathematics or Further Mathematics from the start may choose not to embark on the qualifications. Much will depend on whether the AS level is seen to be a valuable qualification in its own right.

Separately from Ofqual, the A level Content Advisory Board (ALCAB), a private yet government-funded company constituted by the Russell Group, has been tasked with reviewing the content of Mathematics and Further Mathematics A level, through a panel composed mainly of academic mathematicians and maths educators. The government is also consulting separately with the awarding bodies to develop potential models for post-2016 A levels in the two subjects.

Finally, and separately again, the Department for Education is funding a £2.8 million project to inform the A level maths curriculum. This was initially billed as the University of Cambridge designing a new A level curriculum, but in actual fact the university will develop resources to support the existing/reformed A level rather than a new course in its own right. In addition, the university will pilot new approaches in teacher development.

[17] Mark E. Smith, *Independent Chair's report on the review of current GCE 'specification content' within subject criteria: A report to Ofqual* (Coventry: Ofqual, 2013), p.6.

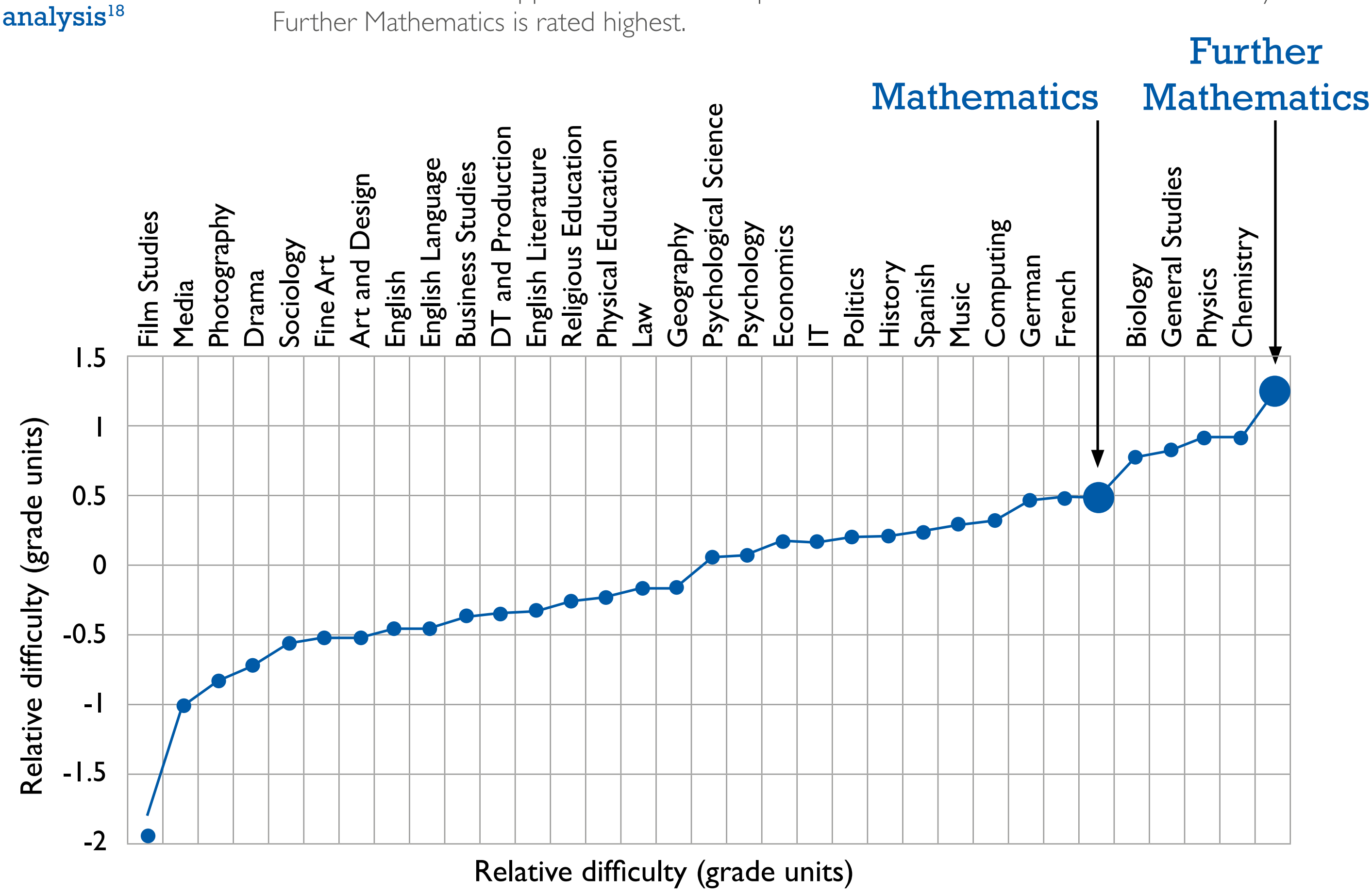
Challenge 3: Supply-side issues: transition from GCSE to A level

The third challenge is the bottleneck in the supply of potential post-GCSE students with sufficient grades and aptitude to feel confident about progressing to A level Mathematics, meaning that this qualification may only ever be accessible to a small minority of the cohort. This would appear to put a natural limit on the numbers taking A level Mathematics.

Students are highly rational in their choice of subjects and particularly in their focus on which ones to drop, since their performance in each domain essentially determines their destination in higher education. A level Mathematics is difficult, and perceived as such. This is reinforced in the guidance that students receive, with those who do not achieve A* or A grades at GCSE Mathematics often being discouraged by teachers - and, where they still exist, careers guidance staff – from choosing A level Mathematics. There is anecdotal evidence that some grade A students are being advised to choose other subjects where they are more likely to get high grades at A level.

Figure 3 shows analysis undertaken by Professor Robert Coe of Durham University, using Rasch analysis on national datasets to explore the relative difficulty of different A level subjects. The model is based on comparing the A level results of individual students taking different subjects, in terms of statistically how "difficult" they find it to obtain a good grade. A level Mathematics appears toward the top end of the scale in terms of relative difficulty and Further Mathematics is rated highest.

Figure 3: Estimate of relative difficulty of A levels using Rasch analysis[18]



[18] Adapted from: Robert Coe and others, *Relative difficulty of examinations in different subjects: Report for SCORE* (Durham: CEM Centre, 2008), p. 87.

As **Figure 4** shows, the vast majority of those choosing AS and A level Mathematics achieved A or A* grades in GCSE Mathematics. Very few students who take the full A level (around 9%) have a grade B in GCSE Mathematics, while almost none have Cs. Virtually all students taking Further Mathematics A level have an A* or A grade in GCSE Mathematics.

Figure 4: Prior GCSE grades for Maths AS/A levels (provisional estimates for students in Year 11 in 2010[19]

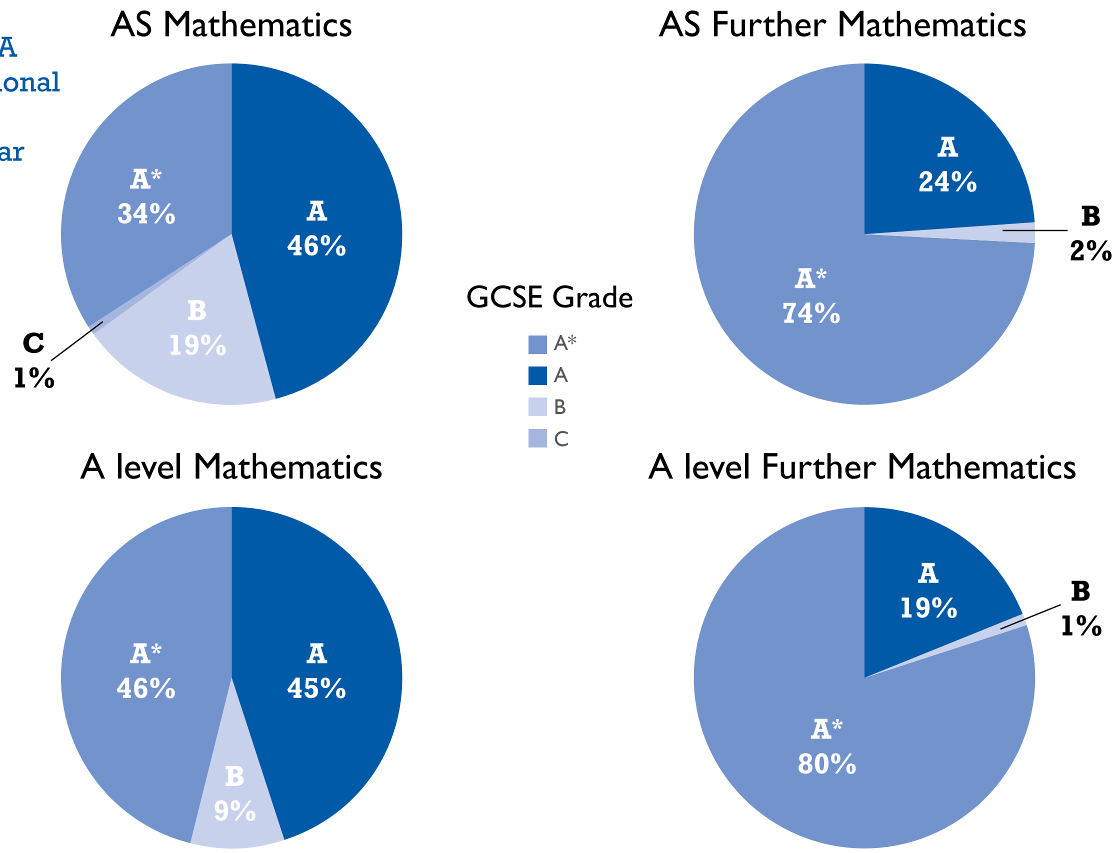
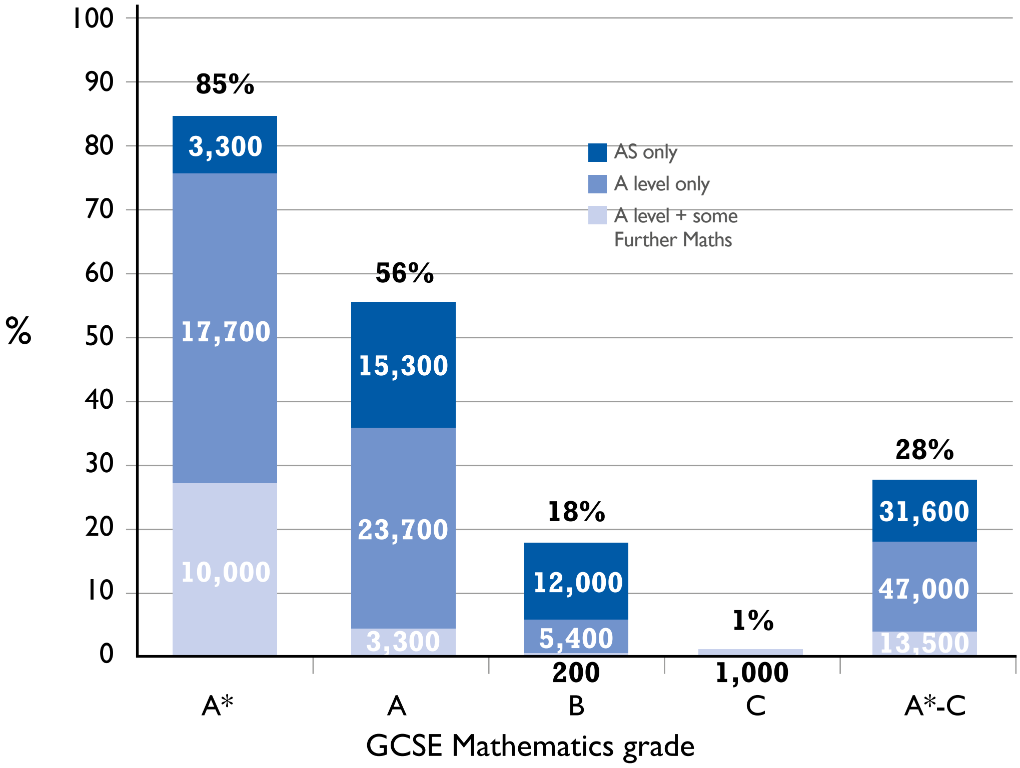


Figure 5 shows the participation statistics from the inverse perspective. Eighty-five per cent of students who achieve an A* at GCSE go on to take the subject to at least AS level. But the proportions drop off sharply as performance at GCSE declines, with only 1-2% of those with GCSE C grades going on to take AS/A level.

[19] Data originates from the National Pupil Database and is for the 2010 cohort of Year 11 students in England. It is preliminary analysis undertaken by Professor Andrew Noyes from the University of Nottingham as part of his study: *Rethinking the Value of A level Mathematics Participation* (unpublished), which is funded by the Nuffield Foundation.

Figure 5: Pupils progressing to Maths AS/A level in England by grade in GCSE Maths (2010)[20]



So, among the higher achievers at GCSE – the A* and A group – many are already choosing A level Mathematics. And students with those two grades make up the overwhelming proportion of those going on to pursue the subject at AS and beyond.

All of the above means the potential to increase A level participation is constrained, and that any toughening-up of GCSE grade boundaries could potentially reduce progression because students may decide they do not have the necessary grades to make a success of the subject post-16.

Table 3 shows how students' GCSE Mathematics grades are strongly linked to their A level Mathematics grades, to a much greater extent than is the case in other subjects. For example, **Table 4** shows that among those achieving a B grade at GCSE in 2008 only 5% went on to gain the top A level grade at the time, an A, with only 20% achieving a B or better. In contrast, in English, 32% of those gaining a B grade at GCSE went on to achieve at least a B at A level, while those awarded Cs in English at GCSE were less likely to end up with an E or U in it at A level than were their counterparts in maths. Yet the range of ability among those taking English A level, as measured by prior GCSE grades, is wider than it is for maths, with students with Bs and Cs in the subject more likely to take it post-16.

[20] Data originates from the National Pupil Database and is for the 2010 cohort of Year 11 students in England. It is preliminary analysis undertaken by Professor Andrew Noyes from the University of Nottingham as part of his study: *Rethinking the Value of A level Mathematics Participation* (unpublished), which is funded by the Nuffield Foundation.

Table 3: Maths A level grade in relation to prior grade at GCSE (2008)[21]

		A level Mathematics grade					
		A	**B**	**C**	**D**	**E**	**U**
Prior GCSE Maths grade	A*	72%	18%	6%	2%	1%	0%
	A	24%	30%	23%	14%	8%	1%
	B	5%	15%	25%	28%	20%	6%
	C	5%	10%	21%	27%	24%	12%

Note: Numbers have been rounded so may not add up to 100%.

Table 4: A level grade in relation to prior grade at GCSE for Mathematics and English[22]

			A level grade					
			A	**B**	**C**	**D**	**E**	**U**
Prior GCSE grade	**Maths**	B	5%	15%	25%	28%	20%	6%
	English	B	6%	26%	41%	24%	3%	0%
	Maths	C	5%	10%	21%	27%	24%	12%
	English	C	1%	7%	30%	42%	18%	1%

Note: Numbers have been rounded so may not add up to 100%.

21 Data provided by the DfE to the Nuffield Foundation in 2011.
22 Data provided by the DfE to the Nuffield Foundation in 2011.

The subjects which students take alongside maths are also worth noting. As **Table 5** shows, a student taking maths is far more likely to be studying physics, chemistry and/or biology than to be pursuing an A level in a social science such as economics or geography. So the A level Mathematics route appears to appeal far more to those students interested in the sciences, which may further constrain the potential for improving post-16 take-up in maths if we concentrate solely on the A level itself. Of course, all students who take science A levels would benefit from advanced maths skills, but the content of the current A level may be designed more around their needs than those of students of other subjects. It is important to ensure that A level Mathematics appeals to non-science students *and* that alternative pathways are available. The goal should be to increase participation in A level Mathematics *and* to develop other routes for those for whom it is not suitable.

Table 5: A level subject combinations taken by students[23]

		who also took A levels in … (for all years 2005-13)								
		Biol	**Chem**	**Econ**	**Geog**	**IT**	**Math**	**Phys**	**Psych**	**Soc**
Proportion of students taking A level in …	**Biology**		0.51	0.03	0.13	0.01	**0.35**	0.11	0.19	0.03
	Chemistry	0.66		0.04	0.08	0.01	**0.56**	0.26	0.09	0.01
	Economics	0.10	0.08		0.14	0.03	**0.43**	0.10	0.08	0.03
	Geography	0.21	0.10	0.08		0.03	**0.17**	0.07	0.12	0.04
	IT	0.07	0.05	0.06	0.09		**0.18**	0.06	0.12	0.06
	Mathematics	**0.29**	**0.36**	**0.12**	**0.08**	**0.03**		**0.33**	**0.07**	**0.01**
	Physics	0.22	0.39	0.07	0.08	0.02	**0.77**		0.02	0.00
	Psychology	0.19	0.07	0.03	0.07	0.02	**0.09**	0.01		0.14
	Sociology	0.06	0.02	0.02	0.06	0.02	**0.03**	0.00	0.29	

Where, then, are the government's current reforms leading? The bottlenecks in increasing take-up of A level Mathematics make the new GCSE Mathematics, to be introduced in 2015, particularly important. The new GCSE is more demanding in its:

- content, with more open-ended problem-solving and advanced coverage of areas such as trigonometry, early calculus and theoretical probability distributions.
- volume, with 20-25% extra teaching time required.

[23] Data originates from the National Pupil Database and is for the GCSE Year 11 cohorts 2003-2011, matched to A level outcomes from 2005-2013 for students in England. It is preliminary analysis undertaken by Professor Andrew Noyes from the University of Nottingham as part of his study: *Rethinking the Value of A level Mathematics Participation* (unpublished), which is funded by the Nuffield Foundation.

- assessment, with modular exams replaced by a terminal one, and with less use of calculators.
- grading and performance measures.[24]

It is also worth considering the relatively low success rates of those who re-take GCSE Mathematics post-16 in the hopes of achieving a better grade. As **Table 6** shows, fewer than one in ten of those who fail to achieve a grade C at 16 go on to achieve one by 19. There has been some improvement in these figures recently, perhaps reflecting significant government attention to the issue, but the Foundation agrees with government concerns that alternatives to re-taking GCSE maths need to be considered.

Table 6: Low success rate on re-taking GCSE Mathematics between 16 and 19

GCSE Mathematics achievement for students in state schools in England

Age 15 in	Cohort size	% achieving A*-C at age 16	% achieving A*-C by age 19	% below grade C at age 16 achieving A*-C by age 19
2006	602,856	53.4	56.5	6.8
2007	600,143	55.6	59.1	7.9
2008	580,155	58.3	61.9	8.6

Challenge 4: Limited success of alternatives to A level mathematics

In the spirit of bringing policy-makers solutions rather than just problems, the Nuffield Foundation's follow-up to *Is the UK an Outlier?* was a more detailed examination of the post-16 mathematics routes on offer in a smaller selection of other countries with high participation rates. One of its key findings was the importance of having more than one high-status route for studying maths. For instance, in New Zealand, a well-respected and increasingly popular option focuses on statistics alongside mathematical applications and fluency.[25] This is obviously a more suitable, and hence more popular, route for those doing social sciences and even some life sciences.

So what are the existing alternatives to A level Mathematics? And how successful have they been in terms of increasing participation?

24 In addition, under changes being introduced to all GCSE subjects, maths grading will move from an alphabetical to a numerical scale. In contrast to changes affecting other subjects, however, maths will retain its system of having papers set according to two overlapping "tiers" of difficulty: foundation and higher. The government's school accountability structure will also be used to incentivise schools to continue to focus on maths, as the subject will have double weighting in the "progress eight". This is an eight-subject performance measure which will be the key to how schools are statistically rated and judged, including through performance tables. Only the grade achieved from a student's first GCSE attempt will count towards school performance, to discourage early and multiple entry. Funding conditions will mean that those who do not achieve a grade C in GCSE Mathematics will continue to study towards GCSE, either through re-takes or courses explicitly aimed at progress towards future GCSE entry and success.

25 Jeremy Hodgen, Rachel Marks and David Pepper, *Towards universal participation in post-16 mathematics: lessons from high-performing countries* (London: Nuffield Foundation, 2013).

Use of Mathematics and other alternatives

First, among the more academic alternatives to A levels, the International Baccalaureate (IB) has been growing steadily in recent years, and it is a requirement of the programme that students study at least one course in mathematics. But the IB still only accounts for 0.5% of the cohort. The newer Cambridge Pre-U, another Level 3 qualification, has an even smaller take-up, and unlike the IB maths is not compulsory. But these options are less about widening participation and more about differentiation at the elite end of the market.

Second, it is important to consider the needs of those with a B or C in GCSE Mathematics, or the minority with A or A* who have not opted for A level. The routes available – or not – to these groups were explored thoroughly in Adrian Smith's 2004 report, *Making Mathematics Count.*[26] This led directly to the development of Use of Mathematics AS/A level (UoM), which built on Free Standing Mathematics Qualifications (FSMQs).

Use of Mathematics AS/A level might seem the obvious central plank of any strategy designed to help realise Michael Gove's aspiration of moving towards universal take-up of the subject post-16. But UoM has not been a success, attracting only tiny numbers of students concentrated in a few institutions. It has never received the necessary backing, and Ofqual is now proposing that it be discontinued, at least partly on the basis that there is too much similar or overlapping content with AS/A level Mathematics.

Why did Use of Mathematics not flourish? There are some possible lessons for any future reform:

1. Any new A level requires significant support from the higher education sector if it is to succeed, but higher education institutions were largely unaware of the existence of UoM.
2. Elements within the mathematics establishment were concerned about the possible impact of UoM on the take-up of Mathematics A level, and were critical of its design and assessment methods.
3. Politically, UoM received inconsistent levels of support from the previous Labour government, followed by opposition from the current administration, both directly, and through the regulator, Ofqual.
4. Provision has never been sufficiently widespread in schools and colleges – and there has been insufficient guidance encouraging students to consider the qualification – to give UoM critical mass.
5. There has been limited marketing from awarding bodies for UoM.
6. Funding and performance measures also, perversely, disincentivised roll-out of UoM in favour of cheaper, more "lightweight" qualifications.

Core Maths

With these lessons in mind, I turn to the key current development: Core Maths, about which the Department for Education has recently published a policy statement.[27]

[26] *Making Mathematics Count, The report of Professor Adrian Smith's Inquiry into Post-14 Mathematics Education* (London: TSO, 2004).
[27] Introduction of 16 to 18 core maths qualifications: Policy Statement (London: DfE, 2014). Available: https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/266717/Policy_statement_on_16-18_Core_Maths_qualifications_-_final__3_.pdf [accessed 20 May 2014].

Its essential characteristics are as follows:

- It is aimed at students who have a C or above at GCSE but who are not currently choosing A level Mathematics. A large proportion of those targeted will have Bs and Cs at GCSE. It would also be an alternative option for the A*/A student, but there would be a danger that too high a take-up from this group might inhibit the growth in AS and A level participation. At the lower end of the prior achievement scale, it is not clear how those failing to get a C grade first time around might progress to Core Maths.
- It will be half an A level in size, but taught over two years to support continued confidence and fluency. The focus will be on the application of mathematics, problem-solving and modelling, mainly using higher tier GCSE Mathematics content.
- Assessment will be predominantly external, rather than coursework/controlled assessment, with something like a pass/merit/distinction structure, resulting in a Level 3 qualification, but not an AS.
- It will be delivered by the existing teaching workforce, with some training provided in the first two years through the £20 million Core Maths Support Programme, which focuses on "early adopter" schools and colleges.
- The awarding bodies are likely to build its design on existing courses, such as AQA's *Use of Mathematics,* OCR's recently-approved *Introduction to Quantitative Methods*, and parts of the International Baccalaureate. The DfE has also funded an investigation into whether another course – *Critical Mathematics* – could be developed based on the ideas of the Cambridge mathematician Timothy Gowers.[28]

The timescale for introduction is rushed, with the first year of teaching effectively being piloted in September 2014. I return to these issues below but note that the Core Maths qualification may have the potential to help those who have not achieved an A in GCSE to continue with maths, and indeed some who have achieved an A but have not chosen the A level. But it is unlikely that this development alone will provide the significant uplift in quantitative skills for a significant part of the student population.

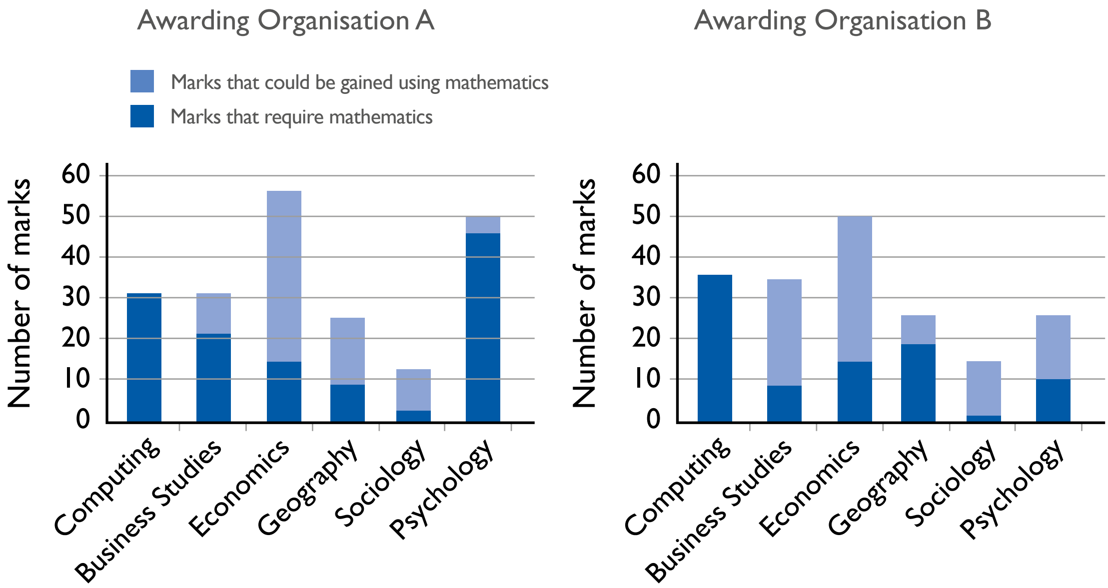
Embedding maths in other subjects

An additional way to improve participation is to support the embedding of maths in other subjects. This would not displace other maths qualifications but reinforce them and show how they can be used to illuminate substantive issues. However the Foundation's study of the extent, difficulty, and type of maths used in the assessment of six other A level subjects found significant disparity between the exam boards.[29] This was compounded by the complexity created by students' ability to choose different modules and questions. For example, two students studying Business Studies could get the same grade, with one using no mathematics at all, and another gaining almost half of their mark from exam questions that require mathematical work. The study concluded that this variation in mathematical content is so great that the qualifications do not give universities or employers a meaningful indication of the level of mathematical skill or understanding of students who have them. (**Figures 6** and **7**).

[28] See: http://www.mei.org.uk/?page=l3_probsolv [accessed 20 May 2014].

[29] *Mathematics in A level assessments: A report on the mathematical content of A level assessments in Business Studies, Computing, Economics, Geography, Psychology and Sociology* (London: Nuffield Foundation, 2012).

Figure 6: Variation in marks that require mathematics and marks that could be gained using mathematics (2010)[30]



Note: Awarding organisations compared are two from among: AQA, CCEA, Edexcel, OCR and WJEC. Data comes from the research project *Mathematics in A level assessments* (2012), although this figure is not included in the report.

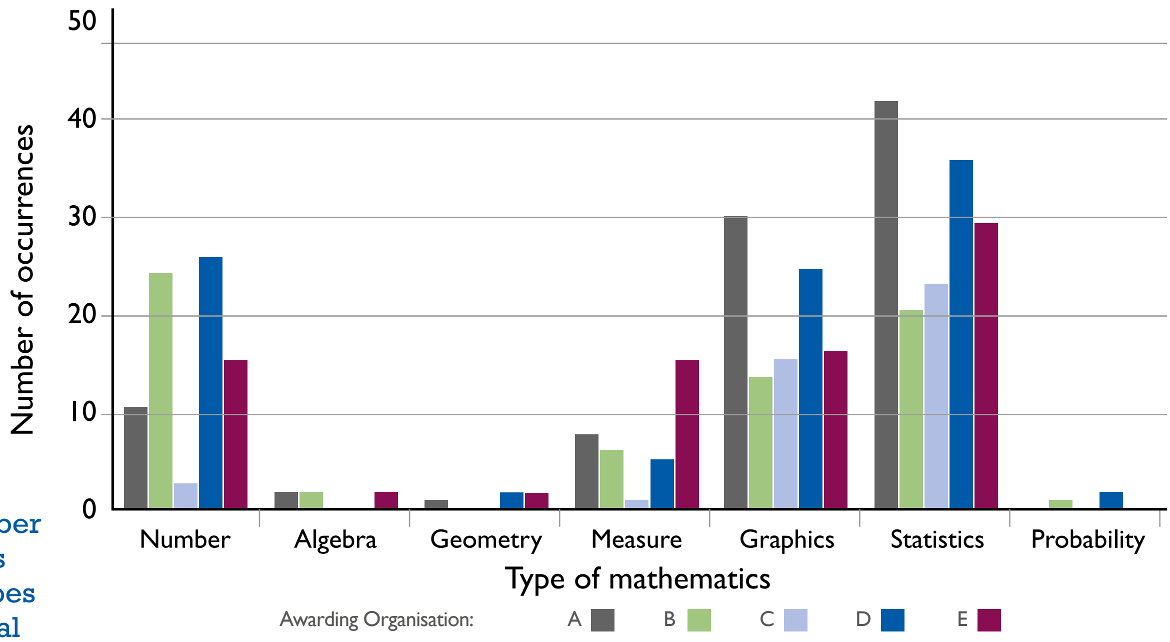


Figure 7: Number of occurrences of different types of mathematical content in AS and A level Geography exams from five different awarding organisations (summer 2010)[31]

Note: Data comes from *Mathematics in A level assessments* (2012). The awarding organisations have been anonymised but include AQA, CCEA, Edexcel, OCR, and WJEC.

[30] Data collated from *Mathematics in A level assessments*.
[31] *Mathematics in A level assessments*, p.39.

Change is on the horizon, however, possibly influenced by this study. Proposals from the DfE and Ofqual for the content and assessment of A levels in a number of subjects explicitly highlight mathematical content, and include minimum weightings for mathematical skills, as shown by **Table 7**. There is as yet no detail on how these would be implemented and monitored. With most subjects needing to be ready for teaching from 2015, time is tight and there is a danger that this requirement could turn into a box-ticking exercise, rather than a significant and creative re-think.

Table 7: Minimum weightings for mathematical skills in A level assessments (for teaching starting September 2015)[32]

At least 10%	Biology, Business, Computing, Geography, Psychology
At least 20%	Chemistry, Economics (with 15% for AS Economics)
At least 40%	Physics

Challenge 5: Demand-side issues from higher education: transition from A level and university

The next challenge centres on the demand from higher education for mathematical skills in the students they recruit and teach. There are three key factors here.

First, there are the demands of higher education courses themselves, which vary considerably across subjects and courses. However, there are common concerns, evidence for which can be seen in reports by the Advisory Committee on Mathematics Education (ACME);[33] the Higher Education Academy,[34] and the Economic and Social Research Council (ESRC).[35] These reports highlight concerns from the higher education sector about a lack of mathematical confidence and fluency in undergraduates, especially among those who have not done any maths for two to three years; a need for statistics beyond the descriptive methods of GCSE, such as inference, experimental design, probability, risk and decision-making; and students' deficiencies in the use of spreadsheets and other types of software. The Higher Education Academy is running a large project examining these issues for a number of disciplines, including sociology and geography.[36]

A second key factor is higher education admission requirements, and more subtle forms of signalling (one of the factors being investigated in the HEA study). Specific subjects such as physics and psychology provide us with some clues as to the emphasis placed on prior maths achievement. In biological sciences, Dr Jenny Koenig's report found that nine out of

[32] *New A level Regulatory Requirements – October 2013.* Available: http://comment.ofqual.gov.uk/a-level-regulatory-requirements-october-2013/ [accessed 20 May 2014].
[33] *Mathematical Needs: Mathematics in the workplace and in Higher Education* (London: ACME, 2011).
[34] Jenny Koenig, *A survey of the mathematics landscape within bioscience undergraduate and postgraduate UK higher education* (Leeds: UK Centre for Bioscience, Higher Education Academy, June 2011).
[35] John MacInnes, *Proposals to support and improve the teaching of quantitative research methods at undergraduate level in the UK* (Edinburgh: University of Edinburgh, 2009).
[36] HEA STEM Strategic Project: Skills in Mathematics and Statistics in the Disciplines and Tackling Transition. Available: http://www.heacademy.ac.uk/resources/detail/stem-project-info [accessed 20 May 2014].

10 undergraduate bioscience degree courses only required GCSE rather than AS or A level maths, with few requiring higher than a C grade at GCSE.[37] The situation is no better in the social sciences outside of economics.

Third, there are the qualifications of undergraduates. **Table 8** shows the proportion of the undergraduate intake with A level Mathematics across 15 different subjects. This increased between 2007 and 2010, reflecting the increased take-up of Mathematics A level across the board. However the proportions for social sciences are still low, for example in sociology the highest figure is still only 5% ; in anthropology it is 10% ; in psychology it is 21% ; and in geography it is around 20%.

The wide variation in students' mathematical backgrounds, as suggested by the figures in this table, has major implications for the design of higher education courses.

Table 8: Proportion of undergraduate intake with A level Mathematics, by degree subject[38]

	with A level Mathematics	
Subject of study at university	**2007**	**2010**
Physics	95%	98%
Economics	59%	68%
General Engineering	62%	64%
Chemistry	55%	64%
Architecture	43%	52%
Computer Science	36%	42%
Biological Sciences (without Psychology)	20%	28%
(Combinations within) Social Studies	20%	24%
Business (all sub-disciplines)	19%	23%
Physical Geographical Sciences	18%	22%
Psychology	16%	21%
Human & Social Geography	14%	18%
Anthropology	9%	10%
Politics	6%	9%
Sociology	3%	5%

[37] Koenig, p.3.
[38] Data based on 2007 and 2010 UCAS data for UK domiciled higher education applicants, as supplied to the Nuffield Foundation by Alice Onion and Professor Brian Follett from the STEM Advisory Forum.

Challenge 6: Mathematics teaching capacity

There is currently a shortage of maths teachers to cover existing levels of post-16 participation and to support existing qualifications. Policies are in place to increase teacher numbers, namely Department for Education bursaries of up to £25,000 including a scholarship system organised by four mathematical organisations.[39] Another government initiative, the Maths and Physics Chairs programme, seeks to increase the supply of maths and physics teachers with high-level subject expertise in non-selective state schools.[40]

These are helpful developments but the scale of the problem is significant. Government evidence to the School Teachers' Review Body (STRB) in December 2013 documents the current shortfall of specialist maths teachers, reporting that 0.7% of maths teaching posts in secondary schools in England were unfilled in 2012. This figure equates to 140 vacancies, the joint highest – along with English – of any subject.[41]

When we consider whether posts are not just filled, but filled by specialist teachers of the subject, the position is even more worrying. The evidence to the STRB was that, as of November 2012, 18% of maths lessons in publicly funded schools in England were taught by non-specialists. An additional 5,500 specialist maths teachers – the highest for any subject – would be needed to make up that shortfall in existing provision, says the submission.

The precise position with regard to the staffing of post-16 maths provision specifically is less clear. But any significant expansion of post-16 participation, whether through existing qualification routes or through new ones, will face an even more serious challenge in terms of teacher recruitment, as supply cannot meet the demand even at current participation levels.

3. Some ways forward

In this concluding section I set out a number of ways to rise to these challenges collectively, including some specific contributions from the Nuffield Foundation.

A push for political consensus.

There is good news in that there are no major political fault lines on these issues. While no party seems to have the appetite to replace A levels in favour of the broader curriculum more common at secondary level in other countries, there is a shared concern about the narrowness of the 16-18 curriculum in an age of mass education.

The coalition government's Core Maths policy is discussed in the previous section. In terms of the opposition, Stephen Twigg, at the time shadow Secretary of State for Education, announced in 2012 the Labour Party's policy of universal participation in maths to the age of 18. More recently, in March 2014, Labour's independent Skills Taskforce published a

39 See: http://www.education.gov.uk/get-into-teaching/subjects-age-groups/teach-maths [accessed 20 May 2014].
40 See: http://www.researchersinschools.org/researchers/maths-and-physics-uplift [accessed 20 May 2014].
41 *Evidence to the STRB: the 2014 pay award*, (London: Department for Education, 2013). Available: https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/268206/DFE-00289-2013.pdf [accessed 20 May 2014].

document on qualifications reform, suggesting all learners continue to study English and maths to 18 as one of the four components of a "National Baccalaureate".[42] However, despite a long section on post-16 maths, ominously there was no mention of Core Maths. So will there be another "Groundhog Day" – of revisiting these issues from scratch – if there is a change of government? This must be avoided at all costs. Reforms should be given the chance to bed down and to be built upon, rather than uprooted and replaced at the first opportunity. Only then will schools and students get consistent signals that quantitative skills matter.

Support for new routes and qualifications from higher education

Evidence from "high-performing" countries suggests that the strongest incentive for students to continue with maths is that they are required to do so to progress to higher education and employment. So there is a clear need for universities to press for new routes and qualifications, to support their development, and to endorse them.

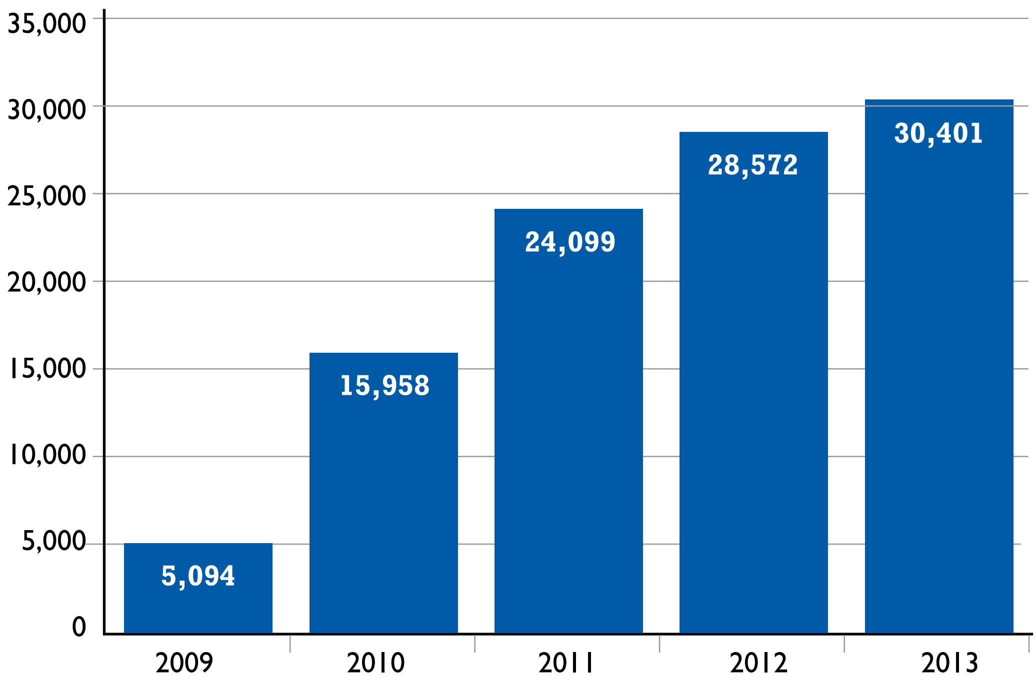
Universities also need to give much more explicit signals as to the mathematical requirements of higher education courses and to the qualifications the courses require for entry – or at least to signal the enhanced prospects of students with these skills.

There is a chicken and egg situation here: higher education institutions won't make post-16 maths a required qualification or even provide a more subtle signal that it is necessary or desirable, because in a competitive market for students to do so would be to rule out large numbers of applicants. So there is still not sufficient incentive for students to choose maths qualifications, particularly in the context described earlier of constraints on the growth in numbers opting for the full A level and changes to the status of the AS level. With the introduction of Core Maths, there is potential to break this log jam, but whether universities will value this qualification if it is not an AS level remains to be seen. Either way, there remains the opportunity for the development of a New Zealand style pathway, not to displace A level Mathematics but to provide an alternative and high level option, with much more focus on statistics and quantitative methods.

The relative success of the Extended Project Qualification (EPQ) may provide some lessons. In the EPQ, students plan, research and execute a project in an area of interest. The qualification is Level 3 and is worth half an A level, requiring 120 guided learning hours. As **Figure 8** shows, the numbers taking the EPQ have grown significantly and rapidly since its introduction, and now stand at roughly 9% of those taking A levels.

[42] *Qualifications matter: improving the curriculum and assessment for all. The third report of the independent Skills Taskforce*, (London: Independent Skills Taskforce, 2014), p. 13. Available: http://www.yourbritain.org.uk/uploads/editor/files/Skills_Taskforce_3rd_report.pdf [accessed 20 May 2014].

Figure 8: Growth in student numbers taking the Extended Project Qualification 2009-2013)[43]



The major factor behind the success of the EPQ is endorsement and practical support from higher education. A wide range of universities, including those that are "research-intensive", have engaged with the EPQ by holding workshops on research methods and providing library access and mentoring. Critically, the EPQ has also been included in the admission offers of many universities, for example by making an offer of ABB plus an A in the EPQ as an alternative to an AAB offer. Imagine the effect of universities offering something similar for AS or Core Maths.

Sharing of content and expertise across sectors and disciplines

In the post-16 phase, there are a number of resources that support the mathematical needs of other subjects. These can be used by teachers of maths or those of other subjects, in either case providing opportunities for cross-curricular teaching and, more importantly, for students to make connections between subjects. Examples include the Nuffield Foundation's Free-Standing Mathematics Activities[44] and MEI's Integrating Mathematical Problem Solving resources.[45] A report from the Royal Statistical Society has also highlighted opportunities for teaching statistics across a range of subjects.[46]

[43] Data from the Joint Council for Qualifications (JCQ). Available: http://www.jcq.org.uk/examination-results/a-levels [accessed 20 May 2014].
[44] See: www.nuffieldfoundation.org/fsma [accessed 20 May 2014].
[45] See: www.mei.org.uk/?section=teachers&page=imps_info [accessed 20 May 2014].
[46] Roger Porkess, *A world full of data: Statistics opportunities across A-level subjects* (London: Royal Statistical Society, 2013). Available: http://www.rss.org.uk/uploadedfiles/userfiles/files/A-world-full-of-data.pdf [accessed 20 May 2014].

At undergraduate level, examples include METAL (Mathematics for Economics: enhancing Teaching and Learning), which provides lecturers and students with free resources,[47] and deSTRESS: Depository of Statistics Resources in Social Sciences.[48]

The Nuffield Foundation, in partnership with the ESRC and the Higher Education Funding Council for England (HEFCE) will be adding to these resources though Q-Step, a £19.5 million programme designed to promote a step-change in quantitative social science training in the UK. Fifteen universities (Q-Step Centres) have received funding to develop and deliver undergraduate programmes, including new courses, work placements, and pathways to postgraduate study. They are sharing expertise and resources across the higher education sector through a support programme, an integral part of which is forging links with schools and employers and increasing the signalling to secondary schools and their students about the importance of quantitative skills in the social sciences.

Input from the higher education sector will also be critical for the assessment of the new Core Maths qualification(s), which will focus on modelling, argumentation and fluency. Finally, many of the topics of interest for Core Maths students, such as quantitative modelling in social sciences, working with big data and mathematical modelling, are best handled using technology such as open source mathematical and statistical packages and data visualisation tools.

Exploration of opportunities and innovations outside the curriculum

There are many ways in which quantitative skills might be promoted and provided beyond curricular classroom study of specific subjects. As an example, the Foundation funds Nuffield Research Placements, which provide A level students on STEM courses the opportunity to work alongside professional researchers for 4-6 weeks in their summer holidays. In recent years we have been successful in widening access to placements to greater numbers of young people from more disadvantaged backgrounds. The vast majority of placements are in the sciences, but we are running a pilot to increase the number of maths-, statistics- and computing-related placements, particularly around the use of quantitative methods in social sciences and industry.

Our strong links with schools give us access to students doing A level Mathematics who may be interested in social sciences. At the moment our efforts are limited by the availability of relevant projects, so we are keen to reach out to new organisations that can offer relevant placements.

Teacher recruitment and development

We have commissioned research to profile the post-16 maths teaching workforce in schools and sixth form and further education colleges and to undertake more detailed modelling in the context of the new qualification routes. This will provide a baseline for new thinking about models of teacher development and recruitment in the longer term. This is likely to be a major and sustained issue for educational policy, and governments will need to prioritise this area if significant growth in post-16 participation is to be achieved.

[47] See: www.metalproject.co.uk/METAL/The%20Project/ [accessed 20 May 2014].
[48] See: www.jisc-content.ac.uk/node/374 [accessed 20 May 2014].

Research into policy and practice

Funding research and development projects is the core of the Nuffield Foundation's work, with post-16 maths being a priority area. We want to support projects which look at quantitative approaches and skills across subjects; transitions into and beyond post-16 maths; use of technology; implications for the school and college workforce; and gender dimensions.[49]

4. Conclusion

There is now widespread acknowledgement of the serious problem of low participation rates in maths across most of the UK. Success in tackling this problem has been limited, and other countries, who already start from a much higher base than in the UK, are not standing still.

A number of potentially significant policy developments may bring more progress. But the challenges outlined in this report are significant, and there is a danger that the complexity and rushed timetable of developments could mean the opportunity for significant structural change is missed.

However, with improved dialogue between the school and higher education sectors, use of high-quality national and international evidence (including that commissioned by the Nuffield Foundation), and the stated support for this agenda from the political parties and from across the maths, science and social science education sectors, there are strong grounds for optimism. One thing is certain: significant progress will require a concerted and long-term plan of action.

[49] See www.nuffieldfoundation.org/mathematics-education for full funding criteria.

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